Spring '07/MAT 117/Exam ~~2~~ 3 **Name:** Solution *Show all your work.*

angle = (relative frequency) $\cdot$ 360° $\quad Z = \dfrac{X-\mu}{\sigma}$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad \sigma = \sqrt{\frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}}$$

$$\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n} \qquad \sigma = \sqrt{\frac{f_1(x_1-\mu)^2 + f_2(x_2-\mu)^2 + \cdots + f_n(x_n-\mu)^2}{f_1 + f_2 + \cdots + f_n}}$$

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1. (9pts) According to the U.S. Bureau of Census, from 1986 to 1995, the percentages of students in grades 10 through 12 who dropped out in a single year were 4.3, 4.1, 4.8, 4.5, 4.0, 4.0, 4.3, 4.2, 5.0, 5.4, given in order of years.

a) Find the median dropout rate.

b) Find the mean dropout rate.

c) Find the standard deviation.

a) put in order: $4.0, 4.0, 4.1, 4.2, \underbrace{4.3, 4.3}_{\text{middle ones}}, 4.5, 4.8, 5.0, 5.4$

avg is $\dfrac{4.3+4.3}{2} = 4.3$

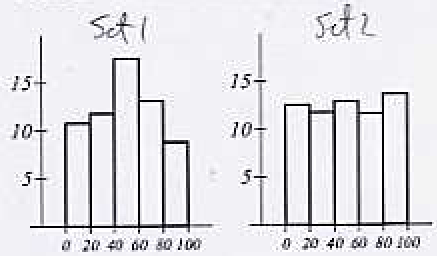
b) $$\mu = \frac{2\cdot 4.0 + 4.1 + 4.2 + 2\cdot 4.3 + 4.5 + 4.8 + 5.0 + 5.4}{10} = \frac{44.6}{10} = 4.46$$

c) $$\sigma^2 = \frac{2(4.0-4.46)^2 + (4.1-4.46)^2 + (4.2-4.46)^2 + 2(4.3-4.46)^2 + (4.5-4.46)^2 + (4.8-4.46)^2 + (5-4.46)^2 + (5.4-4.46)^2}{10}$$

$$= \frac{2(-0.46)^2 + (-0.36)^2 + (-0.26)^2 + 2(0.16)^2 + (0.04)^2 + (0.34)^2 + (0.54)^2 + (0.94)^2}{10}$$

$$= \frac{1.964}{10} = 0.1964 \qquad \sigma = \sqrt{0.1964} \approx 0.4432$$

2. (3pts) Histograms for two data sets, which have the same mean $\mu = 59$, are shown. Which of the data sets will have a greater standard deviation and why?

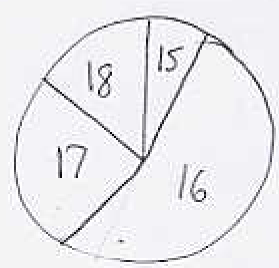


Set 2 will have a greater standard deviation since more data is spread away from μ. In Set 1, data is concentrated around μ.

3. (14pts) The frequency distribution of the minimum age to receive an unrestricted driver's license in each of the 50 states and the District of Columbia is shown in the table.

a) Find the relative frequencies for each class.
b) Find the appropriate angles and draw a pie chart for the data.
c) Find the median of the data.
d) Find the mean of the data.
e) Find the standard deviation of the data.

Minimum Age (yrs)	Number of states	Relative frequency	Angle
15	3	0.0588	21
16	28	0.5490	198
17	12	0.2353	85
18	8	0.1569	56
	51		



c) List is $\underbrace{15, \ldots, 15}_{3}, \underbrace{16, \ldots, 16}_{28}, \underbrace{17, \ldots, 17}_{12}, \underbrace{18, \ldots, 18}_{8}$

middle of 51 is 26th; 26th is here so median = 16

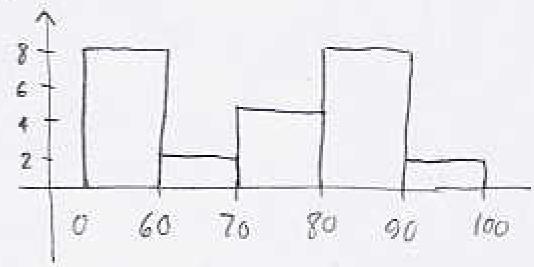
d) $\mu = \dfrac{3\cdot 15 + 28\cdot 16 + 12\cdot 17 + 8\cdot 18}{51} = \dfrac{841}{51} = 16.49$

e) $\sigma^2 = \dfrac{3(15-\mu)^2 + 28(16-\mu)^2 + 12(17-\mu)^2 + 8(18-\mu)^2}{51} = \dfrac{34.745}{51} = 0.68127$

$\sigma = \sqrt{0.68127} = 0.8254$

4. (7pts) This semester on exam 2, my Calculus 1 class achieved scores summarized in the table below. Do the following:
a) Draw a bar graph for the data.
b) Enter a representative value for each interval.
c) Estimate the mean of data.

Range	Frequency	Rep. value
90–100	2	95
80–89	8	84.5
70–79	5	74.5
60–69	2	69.5
0–60	8	30
	25	

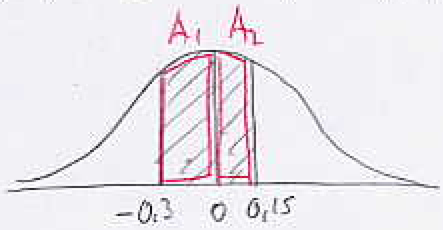


c) $\mu \simeq \dfrac{8\cdot 30 + 2\cdot 64.5 + 5\cdot 74.5 + 8\cdot 84.5 + 2\cdot 95}{25} = \dfrac{1607.5}{25} = 64.3$

5. (10pts) Compute the following probabilities for a standard normal distribution. Draw a picture showing which area you are computing — shading is a good thing!

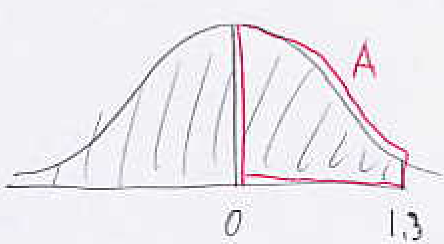
a) $P(-0.3 \le Z < 0.15)$ $= A_1 + A_2 = 0.1179 + 0.0596$

$= 0.1775$



b) $P(Z \le 1.3)$ $= 0.5 + A = 0.5 + 0.4032$

$= 0.9032$



6. (7pts) Based on the U.S. Bureau of the Census statistics, the ages of women who bore a child in 1992 were roughly normally distributed with mean 27.5 years old and a standard deviation of 6 years. Of the women who bore a child in 1992, what is the percentage that were between the ages of 18 and 22?

$$P(18 \le X \le 22) = P\left(\frac{18-27.5}{6} \le Z \le \frac{22-27.5}{6}\right)$$

$$= P(-1.58 \le Z \le -0.92)$$

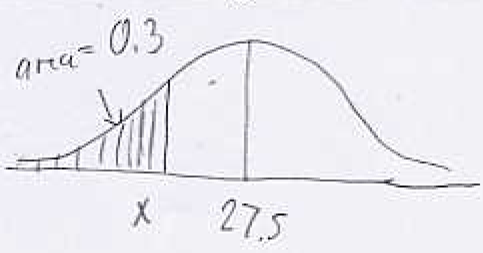
$$= A_2 - A_1$$

$$= 0.4429 - 0.3212$$

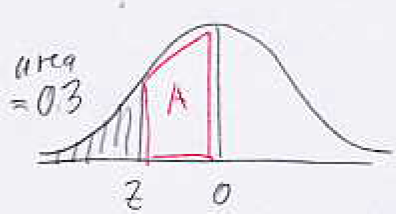
$$= 0.1217$$

8.17 % of women bearing a child fell into this category

Bonus. (5pts) Referring to the above problem, what is the age that falls at the 30th percentile of the ages of the women who bore children in 1992.



convert to standard



$A = 0.2$

Numbers closest to 0.2 are 0.1985 ← closer, corresponds to $z = 0.52$
0.2019

$$\frac{X-27.5}{6} = -0.52$$

since on left side of 0

$$X - 27.5 = -3.12$$

$$X = 27.5 - 3.12 = 24.38 \text{ years}$$

(30% of women were 24.38 years old or younger)