Spring '07/MAT 117/Exam 2 **Name:** Solution *Show all your work.*

$\frac{a}{b} = \frac{1-P(E)}{P(E)}$ $P(E) = \frac{b}{a+b}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

1. (2pts) If 87% of households in a town earn more than \$20,000 in a year, what is the probability that a random selected household earns less than \$20,000 in a year?

$P(\text{not more than } 20{,}000) = 1 - P(\text{more than } 20000)$
$= 1 - 0.87 = 0.13$

2. (2pts) During the first two years of college, you took 80 exams, and on 11 of them there was a question that completely surprised you. What is the empirical probability of receiving a question on an exam that completely surprises you?

$P(\text{complete surprise}) = \frac{11}{80} = 0.1375$

3. (7pts) A coin is tossed three times.
a) List all the equally likely outcomes of this experiment. How many are there?
b) List the outcomes on which exactly one head was tossed.
c) What is the probability of the experiment resulting in exactly one head tossed?

a) HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

there are 8 outcomes

b) HTT
THT
TTH

c) $P(\text{exactly one head}) = \frac{3}{8}$

4. (3pts) If a die is rolled, the odds against getting a 3 or a 5 on the roll are <u>2</u> to <u>1</u>.

4 ways not to get (3 or 5) $\rightarrow \frac{4}{2} = \frac{2}{1}$

2 ways to get (3 or 5) $\rightarrow$

5. (3pts) If the probability of finding a quarter on the sidewalk during a daily walk is 10%, what are the odds against finding a quarter on the sidewalk during a daily walk?

$P(E)=0.1 \qquad \frac{a}{b} = \frac{1-0.1}{0.1} = \frac{0.9}{0.1} = \frac{9}{1}$ odds against are 9 to 1

6. (8pts) You play the following game, for which there is a \$1 charge: if you get a 3 on a roll of a die, you win \$4.50 (and your \$1 is returned), otherwise you win nothing.
a) What is the expected value for this game of chance?
b) If you play the game 25 times, how much do you expect to win (or lose)?
c) Use expected value to determine whether this game represents a fair bet.
d) Use odds to determine whether this game represents a fair bet.

a)

	outcomes	probability
win	4.50	$\frac{1}{6}$
lose	1	$\frac{5}{6}$

expected value $= 4.5 \cdot \frac{1}{6} + (-1) \cdot \frac{5}{6}$

$= \frac{4.5-5}{6} = -\frac{0.5}{6} = -\frac{1}{12}$

expect to lose $\frac{1}{12} \approx -0.0833$ on every game

b) You would expect to lose $25 \cdot \frac{1}{12} = \$2.08$

c) since expected value is negative, the bet is not a fair one.

d) house odds are 4.5 to 1

true odds against are 5 to 1

$\frac{4.5}{1} < \frac{5}{1}$ so it is not a fair bet.

7. (6pts) In a dealer's lot with 35 cars, 27 have power windows, 13 have power seats and 10 have both [redacted] of those features. If a car is randomly selected, what is the probability that
a) it has at least one of the features?
b) it has neither of the features?

a) $P(\underbrace{\text{windows or seats}}_{\text{"at least one feature"}}) = P(\text{windows}) + P(\text{seats}) - P(\text{windows and seats})$

$P(\text{windows or seats}) = \frac{27}{35} + \frac{13}{35} - \frac{10}{35} = \frac{30}{35} \approx 0.8571$

b) $P(\text{not (windows or seats)}) = 1 - \frac{30}{35} = \frac{35}{35} - \frac{30}{35} = \frac{5}{35} = \frac{1}{7} \approx 0.1429$

8. (10pts) On any day at a convenience store, there is a 15% chance that it has run out of "Gong" brand of potato chips, and a 65% chance that at least one of the lights has burned out. Assume these two events are independent.
a) What is the probability that the store has run out of "Gong" potato chips and has at least one light burnt out?
b) What is the probability that the store has all lights working, but has run out of "Gong" potato chips?
c) What is the probability that the store has run out of "Gong" potato chips, given that at least one of the lights has burned out?

a) $P(\text{burned out and "Gong" out}) = P(\text{burned out}) \cdot P(\text{"Gong" out})$

$= 0.15 \cdot 0.65 = 0.0975$

b) $P(\text{not burned out and "Gong" out}) = P(\text{not burned out}) \cdot P(\text{"Gong" out})$

these are independent, too $= 0.35 \cdot 0.15 = 0.0525$

c) $P(\text{"Gong" out} \mid \text{burned out}) = P(\text{"Gong" out}) = 0.15$

since these events are independent.

32 children

9. (9pts) Two children are chosen at random from a group of 15 boys and 17 girls.
a) What is the probability that both children are boys?
b) What is the probability that the second child is a girl?
c) What is the probability that the second child is a girl, given that the first child was a girl?

a) P(1st boy and 2nd boy) = P(1st boy) · P(2nd boy | 1st boy)

$$= \frac{15}{32} \cdot \frac{14}{31} = 0.2117$$

b) P(2nd is girl) = $\frac{17}{32}$ (as though we picked just one)

≈ 0.5313

c) P(2nd girl | 1st girl) = $\frac{16}{31} \approx 0.5161$

Bonus. (5pts) Jorge and Manuela are separately asked to choose between strawberries and kiwifruit. They will get a snack only if they name different choices. If Jorge names strawberries 70% of the time and kiwifruit 30% of the time, and Manuela names strawberries 65% of the time and kiwifruit 35% of the time, what is the probability that they get a snack?

P(get snack) = P((Jorge strawberry and Manuela kiwifruit) or (Jorge kiwifruit and Manuela strawberry))

[mutually exclusive events] = $\underbrace{\text{P(J. strawberry and M. kiwifruit)}}_{\text{independent events}}$ + $\underbrace{\text{P(J. kiwifruit and Manuela straw.)}}_{\text{independent events}}$

= P(J. strawberry) · P(M. kiwifruit) + P(J. kiwifruit) · P(M. strawberry)

$= 0.70 \cdot 0.35 + 0.30 \cdot 0.65 = 0.245 + 0.195 = 0.44$

44% chance they get snack.