Spring '07/MAT 117/Exam 1 **Name:** Solution *Show all your work.*

$F = P(1+rt)$ $F = P\left(1+\frac{r}{n}\right)^{nt}$ $F = D\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$ $P = R\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $APY = \left(1+\frac{r}{n}\right)^{n}-1$

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1. (4pts) Lou deposits a certain amount of money in an account bearing 4.23% simple interest. After 8 months he withdraws \$462.69. How much did he deposit?

$F = P(1+rt)$

$462.69 = P\left(1+0.0423\cdot\frac{8}{12}\right)$

$462.69 = P\cdot 1.0282 \quad | \div 1.0282$

$P = \frac{462.69}{1.0282} = 450$

2. (5pts) True story: a short-term loan company advertises on its website that one can get a \$400 loan from them that is repaid after 14 days with \$470. What simple annual interest rate are they charging?

$F = P(1+rt)$

$470 = 400\left(1+r\cdot\frac{14}{365}\right) \quad | \div 400$

$\frac{470}{400} = 1 + r\cdot 0.0383\ldots \quad | -1$

$1.175 - 1 = r\cdot 0.0383\ldots \quad | \div 0.0383\ldots$

$\frac{0.175}{0.0383\ldots} = r$

$r = 4.5625$, which is 456.25% yikes!

3. (6pts) What is a better deal on a certificate of deposit:
a) an account earning 3.17%, compounded weekly, or
b) an account earning 3.15%, compounded daily?

$F = P\left(1+\frac{r}{n}\right)^{nt}$

a) $F = 1\cdot\left(1+\frac{0.0317}{52}\right)^{52} = (1.000609\ldots)^{52} = 1.032197827 \leftarrow$ more, so a better deal

b) $F = 1\cdot\left(1+\frac{0.0315}{365}\right)^{365} = (1.000086\ldots)^{365} = 1.031999573$

L enough to see what happens to a deposit of \$1.

4. (6pts) On February 5th, 1997 the stock of Pepsico, Inc. closed at \$25.26 per share. On February 5th, 2007 it closed at \$64.83 per share. Find the annual compound interest rate that this growth corresponds to.

$$F = P\left(1+\frac{r}{n}\right)^{nt}, \quad n=1$$

$$64.83 = 25.26\left(1+\frac{r}{1}\right)^{1\cdot 10} \quad \Big| \div 25.26$$

$$2.566\ldots = (1+r)^{10} \quad \Big| \wedge \tfrac{1}{10}$$

$$(2.566\ldots)^{\frac{1}{10}} = \left((1+r)^{\cancel{10}}\right)^{\cancel{\frac{1}{10}}}$$

$$1.0988\ldots = 1+r \quad |-1$$

$$r = 0.0988395\ldots$$

$$r \approx 9.88\%$$

5. (6pts) Barack would like to use some of his own money to finance a political campaign. How much should he deposit weekly into an account bearing 5%, compounded weekly, if he would like to have \$1,000,000 in a year-and-a-half?

$$F = D\,\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$$

$$1{,}000{,}000 = D\,\frac{\left(1+\frac{0.05}{52}\right)^{52\cdot 1.5}-1}{\frac{0.05}{52}}$$

$$1{,}000{,}000 = D\,\frac{(1.000961\ldots)^{78}-1}{0.000961\ldots}$$

$$1{,}000{,}000 = D\cdot 80.959\ldots \quad \Big| \div 80.959$$

$$D = \frac{1{,}000{,}000}{80.959\ldots} \approx 12{,}351.91 \text{ weekly}$$

6. (15pts) PC and Mac have spent a lot of time together lately, so they decided to jointly buy a plasma TV. The biggest they could find was a 103-inch retailing for \$70,000 (I kid you not!), for which they have secured a 5-year loan at 8.49%, compounded monthly.

a) What is their montly payment on the loan?
b) How much do they owe after 4 years?
c) What are their total payments over the course of the loan?
d) Which portion of their 1st payment goes toward interest, and which towards the principal?

a) $P = R\,\dfrac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$

$70{,}000 = R\,\dfrac{1-\left(1+\frac{0.0849}{12}\right)^{-12\cdot 5}}{\frac{0.0849}{12}}$

$70{,}000 = R\,\dfrac{1-(1.007075)^{-60}}{0.007075}$

$70{,}000 = R \cdot 48.75\ldots$

$R = \dfrac{70000}{48.75\ldots} = 1435.82$

b) debt after 4 years = present value of payments for remainder of term (1 year)

$P = 1435.82\,\dfrac{1-(1.007075)^{-12\cdot 1}}{0.007075}$

$= 1435.82 \cdot 11.465\ldots$

$= 16{,}462.97$

c) $1435.82 \cdot 60$ payments

$= 86{,}149.20$

d) 1st payment:

interest $= 70{,}000 \cdot \dfrac{0.0849}{12}$

$= 495.25$

Toward principal:

$1435.82 - 495.25 = 940.57$

7. (8pts) If you deposit \$400 every quarter in an account bearing 7.26%, compounded quarterly, how long will it take until you have \$10,000 in the account?

$$F = D\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$$

$$10000 = 400\frac{\left(1+\frac{0.0726}{4}\right)^{4t}-1}{\frac{0.0726}{4}} \quad \Big| \div 400$$

$$25 = \frac{1.01815^{4t}-1}{0.01815} \quad \Big| \cdot 0.01815$$

$$0.45375 = 1.01815^{4t}-1 \quad \Big| +1$$

$$1.45375 = 1.01815^{4t} \quad \Big| \log$$

$$\log 1.45375 = \log 1.01815^{4t}$$

$$\log 1.45375 = 4t \log 1.01815 \quad \Big| \div 4\log 1.01815$$

$$t = \frac{\log 1.45375}{4\log 1.01815} = 5.20 \text{ years}$$

Bonus. (5pts) A couple of newlyweds took out a 15-year, \$234,000 loan to finance their new home. The interest rate on this loan is 5.73% compounded monthly, making their monthly payment \$1940.65. How long will it be until they owe half the amount on the loan? *Hint: only one formula is needed.*

$$F = R\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$$

$$117{,}000 = 1940.65\frac{1-\left(1+\frac{0.0573}{12}\right)^{-12t}}{\frac{0.0573}{12}} \quad \Big| \div 1940.65$$

$$60.28.. = \frac{1-(1.004775)^{-12t}}{0.004775} \quad \Big| \times 0.004775$$

$$0.28788.. = 1 - 1.004775^{-12t} \quad \Big| -1$$

$$-0.7121.. = -1.004775^{-12t} \quad \Big| \cdot(-1)$$

$$0.7121.. = 1.004775^{-12t} \quad \Big| \log$$

$$\log 0.7121.. = \log 1.004775^{-12t}$$

$$\log 0.7121.. = -12t \log 1.004775$$

$$t = \frac{\log 0.7121}{-12\log 1.004775} = 5.939$$

this is how many years remain, so after

$$15 - 5.939.. = 9.06 \text{ years}$$