

5. (5pts) A werewolf is moving in the xy -plane. At the moment when he is at point $(3, 2)$ the rates of change of his x and y coordinates are $\frac{dx}{dt} = -1$, $\frac{dy}{dt} = 3$. What is the rate of change of his distance to the origin?

6. (8pts) The function $f(x, y, z) = z - x^2 - y^2$ is given.

a) Draw the level surfaces for levels 0, 2 and 5.

b) At point $(1, -2, 3)$, in which direction is the directional derivative the greatest and how much is it?

c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = -2x\mathbf{i} - 2y\mathbf{j} + \mathbf{k}$ and C is a path from any point on the level surface $z - x^2 - y^2 = 9$ to any point on the level surface $z - x^2 - y^2 = 2$.

7. (7pts) Find the local maximum and minimum values and saddle point(s) of the function $f(x, y) = 4y - x^2 - y^2$.

8. (7pts) Find $\iint_D x \, dA$ if D is the part of the region bounded by the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 9$ that is above the line $y = -x$.

9. (7pts) Use either cylindrical or spherical coordinates to set up $\iiint_E x^2 + z^2 dV$ where E is the region inside the sphere $x^2 + y^2 + z^2 = 16$ and between the planes $y = 0$ and $y = \sqrt{3}x$. Do not evaluate the integral.

10. (8pts) Sketch the region of integration and give the two integrals that end in $dx dz dy$ and $dy dz dx$ that are equivalent to the integral $\int_0^1 \int_{1-y^2}^1 \int_0^{1-x} f dz dx dy$.

11. (10pts) Let D be the region between the parabola $y = x^2$ and the line $y = 9$ and let C be the boundary of this region, oriented clockwise. Evaluate the integral $\int_C x^2 dx + 2xy dy$ in two ways:

a) directly

b) using Green's theorem.

12. (8pts) Set up the double integral for $\iint_S yz dS$ where the surface S is the part of the cylinder $x^2 + y^2 = 4$ between the planes $z = 1$ and $z = 5$. Carry out the set-up until you get iterated single integrals, but do not evaluate the integral.

13. (5pts) Is the field $\mathbf{F}(x, y) = ye^x\mathbf{i} + (e^x + 2y)\mathbf{j}$ conservative? If it is, find the function f so that $\nabla f = \mathbf{F}$.

Bonus 1. (4pts) Use the divergence theorem to compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$ if S is the boundary of the cube bounded by the planes $x = \pm 1$, $y = \pm 2$, $z = \pm 3$ and $\mathbf{F} = 3x\mathbf{i} + xy\mathbf{j} + 2xz\mathbf{k}$.

Bonus 2. (4pts) Let $\mathbf{F} = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ and let C be the boundary of the planar region D on which \mathbf{F} is defined. Apply Stokes' theorem to the surface D in order to get Green's theorem. (In other words, Green's theorem is a special case of Stokes' theorem when the surface is a planar region.)