Fall '03/MAT 309/Exam 2 **Name:** *Show all your work.*

1. (5pts) Sketch the domain of the function $\dfrac{\ln(9-x^2-y^2)}{\sqrt{y-x-1}}$.

2. (5pts) Find all the partial derivatives of $f(x,y,z) = xe^{yz}\cos(xz)$.

3. (6pts) If $z = \sqrt{x^2+y^2}$ and $x = \dfrac{s}{t}$, $y = s2^t$ find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

4. (5pts) The radius r and height h of a cylinder are changing with time. At what rate is the volume of the cylinder changing at the moment when $r = 2m$, $h = 5m$, r is increasing at rate 0.02m/s and h is decreasing at rate 0.03m/s?

5. (4pts) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 9$ at the point $(1, -2, 2)$.

6. (5pts) Show that $\lim_{(x,y)\to(0,0)} \dfrac{xy + y^2}{x^2 + y^2}$ does not exist.

7. (7pts) Consider the function $f(x, y, z) = \dfrac{x}{y + 3z}$ at the point $(2, 2, -1)$.
a) Find the directional derivative of f in direction of vector $2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$.
b) In which direction is the directional derivative the greatest and how much is it?
c) In which directions is the directional derivative 0?

8. (8pts) Find three positive numbers whose sum is 15 and whose product is the greatest possible. Use the criterion involving $D(x, y)$ to check that the product is indeed maximal at the point you find. Note that this is a two-variable problem.

9. (5pts) Rock fan Lester is at the concert of the heavy-metal band "Chop Sound". The concert is taking place in the upper half-plane $\{(x, y)|y \geq 0\}$, Lester is currently at point (4,1) and the loudness of music is given by the function $L(x, y) = \dfrac{y}{x^2}$.

a) Draw level curves of $L(x, y)$ for $k = 0, \frac{1}{16}, \frac{1}{2}, 1$.

b) Lester can hardly hear anything where he is. Show the path he has to take if he follows the direction of greatest increase of loudness at every point.

Bonus. (5pts) Referring to above problem, show that Lester will travel along the curve $\dfrac{x^2}{18} + \dfrac{y^2}{9} = 1$. In other words, show that this curve has the property that its tangent vector at any point is always parallel to the direction of greatest increase of L at that point. Hint: the curve has the parametrization $x = 3\sqrt{2}\cos t$, $y = 3\sin t$.