

1. (5pts) If \mathbf{a} and \mathbf{b} are vectors sketched below, draw the vectors $-2\mathbf{a}$, $3\mathbf{a} + 2\mathbf{b}$ and $\mathbf{b} - 2\mathbf{a}$.

2. (7pts) The triangle ABC is given by its vertices $A(6, 3, -3)$, $B(4, 0, 1)$ and $C(5, -2, 0)$.

a) Find the angle at vertex A .

b) Is it a right triangle?

3. (5pts) The cylindrical coordinates of a point are $(\sqrt{3}, \frac{5\pi}{4}, 1)$.

a) Sketch the point.

b) Find the spherical and cartesian coordinates of the point (exact numbers, not decimal).

4. (7pts) This problem is about the surface $y^2 + 4z^2 - x^2 = 1$.

a) Sketch and identify the traces of this surface in the coordinate planes.

b) Sketch and identify the surface in 3D.

5. (7pts) The parametric curve $x = 3t$, $y = 2 \cos t$, $z = 2 \sin t$ is given.

a) Sketch the curve and identify it.

b) Reparametrize the curve with respect to arc length measured from the point where $t = 0$ in the direction of increasing t .

6. (9pts) Acceleration of a rocket-propelled tomato is given by $\mathbf{a}(t) = 15\sqrt{t}\mathbf{j} + 2t\mathbf{k}$.

a) Find the position of the tomato at time t if $\mathbf{v}(1) = 2\mathbf{i} + 10\mathbf{j} + \mathbf{k}$ and $\mathbf{r}(1) = \mathbf{i} - \mathbf{k}$.

b) When does it pass through the xy -plane?

7. (10pts) The lines $x = 2 + 3t$, $y = 1 - t$, $z = 1 + 2t$ and $\frac{x-1}{-6} = \frac{y}{2} = \frac{z+1}{-4}$ are given.

a) Explain why the lines are parallel.

b) Find the equation of the plane determined by the two lines.

c) Find the equation of the plane parallel to the plane in b) that passes through the point $(1, 6, 3)$.

Bonus. (5pts) Show that $\frac{d}{dt}|\mathbf{r}(t)| = \frac{1}{|\mathbf{r}(t)|}\mathbf{r}(t) \cdot \mathbf{r}'(t)$. Hint: Set $f(t) = |\mathbf{r}(t)|$, notice that $f(t)^2 = \mathbf{r}(t) \cdot \mathbf{r}(t)$ and differentiate away. Do not use coordinates unless you want the inelegant solution prize.