1. (11pts) Use formulas to expand:
a) $(x+2 z)^{2}=$
b) $(5 x-7)(5 x+7)=$
c) $\left(x^{3}+4 y\right)^{2}=$
d) $(x-2)^{3}=$
2. (8pts) Factor the following. Use either a known formula or a factoring method.
a) $x^{2}+3 x-28=$
b) $6 x^{2}+7 x-5=$
c) $x^{3}+64=$
3. (3pts) Verify the formula for the difference of cubes by multiplying out:
$(x-a)\left(x^{2}+x a+a^{2}\right)=$
4. (8pts) Simplify.
a) $\frac{3 x+7}{x^{2}+x-20}-\frac{x+1}{x^{2}+5 x}=$
b) $\frac{6-\frac{x-1}{x+4}}{2 x+\frac{x-5}{x+4}}=$
5. $(2 \mathrm{pts})$ Solve the equation: $2(x-5)+7=5-3 x$
6. (3pts) Solve for $x: c x-c=d x+d$
7. (6pts) The points $A(3,-4), B(1,5), C(-4,0)$ are given.
a) Draw the points in the coordinate system.
b) Which of the points $B$ and $C$ is closer to $A$ ? Make a guess by looking at the picture and then determine algebraically.
8. (7pts) Solve the following equations:
a) $x^{2}-2 x=15$
b) $3 x^{2}=17 x+28$
9. (6pts) The price of a dress was reduced twice: during September its cost was $25 \%$ below the original price and during October it was $40 \%$ below the original price. If the difference of the September and October prices was $\$ 39$, what was the original price?
10. (6pts) Farmer Jerry wishes to enclose a rectangular field with a fence. If the length of the field is to be 2 miles more than the width, and the diagonal is to be 5 miles, what should the dimensions of the field be?
11. (8pts) Put the following expressions into standard form $a+b i$ :
a) $(3-4 i)[2+i(1+5 i)]=$
b) $\frac{7-3 i}{4+i}=$
c) (justify also) $i^{99}=$

Solve the equations algebraically:
2. $(5 \mathrm{pts}) x^{4}+9 x^{2}-22=0$
3. $(5 \mathrm{pts}) ~ x+3=\sqrt{2 x+41}$
4. (6pts) How much pure water should be added to 30 ml of a $30 \%$ solution of muriatic acid to get a $12 \%$ solution?
5. (6pts) A bathroom tub fills in 15 minutes if the faucet is open and the stopper is in place. With the faucet closed and the stopper removed, the tub empties in 20 minutes. How long does it take to fill the tub if the faucet is open and the stopper is removed? (Hint: think of faucet and stopper as people contributing to or taking away from a job.)

1. (4pts) Let $f$ be given by $f(x)=\frac{2 x+3}{3 x-7}$. Find the following for this function: $f(7 t)$, $f(x-1)$. (Simplify where possible).
2. (4pts) Find the domain of $f(x)=\frac{2 x-5}{\sqrt{8-3 x}}$.
3. (9pts) Use the graph of the function $f$ at right to answer the following questions.
a) What is the domain of $f$ ?
b) What is the range of $f$ ?
c) Find $f(3)$ and $f(-2)$.
d) Where is the function decreasing?
e) What are the solutions of the equation $f(x)=3$ ?
f) Where is $f(x)<0$ ?

4. (9pts) The function $f(x)=x^{4}-7 x^{2}+6 x+7$ is given.
a) Sketch the graph of $f$ on paper.
b) List the numbers where $f$ has a local minimum or maximum. What are the local minima and maxima (i.e. the $y$-values)? Accuracy: 4 decimal points.
c) List the intervals where $f$ is decreasing.
d) What is the range of this function?
5. (4pts) The function $f(x)=x^{3}-3 x+7$ is given.
a) Determine algebraically whether this function is even, odd or neither.
b) Graph the function on paper. Does the graph support your conclusion from a) and why?
6. (8pts) A customer with Rushy-Rushy cab company found that on one occasion they paid $\$ 5.75$ to ride 2 miles, and on another they paid $\$ 10.75$ to ride a cab for 5 miles.
a) Find the cost of riding a cab as a function of miles traveled, assuming it is linear.
b) How far can a customer get if they have $\$ 20$ in their pocket?
7. (5pts) Use the basic graph of $y=|x|$ and transformations to help you sketch the graph of $y=\frac{1}{2}|x|+4$. Explain how you transform the original graph and what the axis of symmetry of the new graph is.
8. (8pts) The graph of the function $f$ is given below. On three separate graphs, sketch the graphs of the functions $f(x-3),-2 f(x)$ and $f(-x)+4$. Label all the relevant points.

9. (9pts) The Commonwealth of Norlandia, whose currency is Norlandisk Kroner (NK), assesses income tax based on the rules below.
a) Compute tax on taxable amounts of NK 3,000, NK 4,700 and NK 16,500.
b) Write the (piecewise defined) function that computes the income tax $T(x)$ as a function of taxable amount $x$.
c) Graph the function $T$.

| If taxable amount is: | Tax is: |
| :---: | :---: |
| NK 4,000 or less | $10 \%$ of taxable amount |
| Over NK 4,000 but not over NK 10,000 | NK 400 plus $15 \%$ of amount over NK 4,000 |
| Over NK 10,000 | NK 1300 plus 25\% of amount over NK 10,000 |

1. (4pts) The table below indicates the values of $f(x)$ and $g(x)$ for certain numbers. Find the requested composites at right.

$$
\begin{aligned}
& (f \circ g)(1)= \\
& (g \circ f)(5)= \\
& (f \circ f)(3)= \\
& (g \circ g)(-1)=
\end{aligned}
$$

| x | -3 | -1 | 1 | 3 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(\mathrm{x})$ | -1 | 5 | 3 | 7 | -3 |
| $\mathrm{~g}(\mathrm{x})$ | 3 | -3 | 5 | -1 | 1 |

2. (8pts) Let $f(x)=x^{2}-5 x+2$ and $g(x)=\sqrt{x-5}$. Find the following composites and simplify where possible:
$(f \circ g)(x)=$
$(g \circ f)(x)=$
$(g \circ g)(x)=$
3. (4pts) Find functions $f$ and $g$ so that $f \circ g=H$, if $H(x)=(x-3)^{2}$. Find two different solutions to this problem, neither of which is the "stupid" one.
4. (5pts) The graph of a function $f$ is given. Use it to find the graph of $f^{-1}$, labeling the relevant points and showing its asymptote.

5. (5pts) Find the inverse of $g(x)=\frac{3 x-1}{4 x+7}$ and the range of $g$.
6. (4pts) Use the basic graph of $y=e^{x}$ and transformations to help you sketch the graph of $y=3-e^{x}$. Explain how you transform the original graph and what the asymptote of the new graph is.
7. (4pts) Evaluate without using the calculator:
$\log _{6} 216=\quad \log _{4} \frac{1}{64}=\quad \log _{3} \sqrt{27}=\quad \log _{a} \frac{1}{\sqrt[4]{a}}=$
8. (6pts) Solve the equations:
$\log _{x+1} 125=3$
$17^{2 x+5}=4$
9. (4pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.
$\log _{4}\left(64^{2 x-2} \cdot \sqrt[4]{\left(x^{2}-x\right)^{5}}\right)=$
10. (4pts) Write the following as a single logarithm. Simplify if possible.
$\frac{3}{4} \log \left(u^{2} v\right)^{4}-2 \log \left(u^{3} v^{7}\right)=$
11. (1pts) Compute the following number using your calculator. Show how you obtained your number.
$\log _{5} 24=$
12. ( 5 pts ) Solve the equation:
$\log _{6}(9-x)+\log _{2}(x+3)=2$
13. (6pts) In 1999, Spurt City was recorded to have 10,000 residents. In 2007, records show it had 18,000 residents. Do the following, assuming the growth was exponential.
a) Find the growth rate. Write the function that expresses the population $t$ years from 1999. b) Find the population in 2005 .
14. (8pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.
$\sin 225^{\circ}=$
$\tan \frac{3 \pi}{2}=$
$\cos \left(-\frac{2 \pi}{3}\right)=$
$\cot 120^{\circ}=$
15. (4pts) Use the picture below to estimate $\sin \frac{5 \pi}{8}$ and $\cos \frac{5 \pi}{8}$. Then evaluate with a calculator and compare the results.

16. (5pts) If $\sin \theta=-\frac{\sqrt{4}}{9}$ and $\theta$ is in the fourth quadrant, find $\cos \theta, \cot \theta, \sec \theta$. Draw a picture.
17. (5pts) Draw two periods of the graph of $y=-2 \sin (3 x)$. What is the amplitude? The period? Indicate where the special points are ( $x$-intercepts, peaks, valleys).
18. (5pts) You are standing on the ground, away from a building, and are holding a taut string that is attached to the top of the building. If the length of the string is 125 ft and the angle of elevation of the string is $73^{\circ}$, how tall is the building?
19. (3pts) Use trigonometric identities to simplify without using the calculator:
$\csc ^{2} 53^{\circ} \cos ^{2} 53^{\circ}-\sec ^{2} 37^{\circ}=$
20. (8pts) Without using the calculator, find the exact values of the following inverse trigonometric functions. Draw the unit circle and the appropriate angle. $\arccos 0=\quad \arcsin \frac{\sqrt{3}}{2}=\quad \arctan (-1)=\quad \arccos \left(-\frac{1}{2}\right)=$
21. (5pts) Find the exact values of the expressions below. Use a picture if necessary, but do not use the calculator.
$\tan (\arctan 37)=$
$\arcsin \left(\sin \left(\frac{4 \pi}{5}\right)\right)=$
22. (4pts) Find the exact value of the expression below. Draw a picture and do not use the calculator.
$\cot \left(\arccos \left(-\frac{2}{7}\right)\right)=$
23. (4pts) Use the picture below to estimate (in degrees) $\arccos (-0.8)$ and $\arctan (1.2)$. Then evaluate these numbers using a calculator and compare your answers.

24. (4pts) Solve for $\theta$ in $[-\pi, \pi]$.
$2 \sin \theta+\sqrt{3}=0$
25. $(5 \mathrm{pts})$ Find all the solutions of $\cos (5 \theta)=\frac{1}{2}$.
