

1. (4pts) Evaluate without using the calculator:

$$\log_6 216 = 3$$

$$6^3 = 216$$

$$\log_4 \frac{1}{64} = -3$$

$$4^3 = 64$$

$$4^{-3} = \frac{1}{64}$$

$$\log_3 \sqrt{27} = \frac{3}{2}$$

$$\sqrt{27} = \sqrt{3^3} = 3^{\frac{3}{2}}$$

$$\log_a \frac{1}{\sqrt[4]{a}} = -\frac{1}{4}$$

$$\frac{1}{\sqrt[4]{a}} = \frac{1}{a^{\frac{1}{4}}} = a^{-\frac{1}{4}}$$

2. (6pts) Solve the equations:

$$\log_{x+1} 125 = 3$$

$$(x+1)^3 = 125 \quad | \sqrt[3]{\phantom{x}}$$

$$x+1 = 5$$

$$x = 4$$

$$17^{2x+5} = 4 \quad | \ln$$

$$\ln 17^{2x+5} = \ln 4$$

$$(2x+5) \ln 17 = \ln 4 \quad | + \ln 17$$

$$2x+5 = \frac{\ln 4}{\ln 17}$$

$$2x = \frac{\ln 4}{\ln 17} - 5$$

$$x = \frac{\frac{\ln 4}{\ln 17} - 5}{2} \approx$$

$$-2.2553$$

3. (4pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\begin{aligned} \log_4 (64^{2x-2} \cdot \sqrt[5]{(x^2-x)^5}) &= \log_4 64^{2x-2} + \log_4 (x^2-x)^{\frac{5}{4}} \\ &= (2x-2) \underbrace{\log_4 64}_{=3} + \frac{5}{4} \log_4 (x^2-x) \\ &= 6x-6 + \frac{5}{4} \log_4 (x^2-x) \end{aligned}$$

4. (4pts) Write the following as a single logarithm. Simplify if possible.

$$\begin{aligned} \frac{3}{4} \log(u^2v)^4 - 2 \log(u^3v^7) &= \log \left( (u^2v)^4 \right)^{\frac{3}{4}} - \log (u^3v^7)^2 \\ &= \log \frac{(u^2v)^{4 \cdot \frac{3}{4}}}{(u^3v^7)^2} = \log \frac{(u^2v)^3}{(u^3v^7)^2} \end{aligned}$$

$$= \log \frac{u^6 v^3}{u^6 v^{14}} = \log \frac{1}{v^{11}} = \log v^{-11} = -11 \log v$$

5. (1pts) Compute the following number using your calculator. Show how you obtained your number.

$$\log_5 24 = \frac{\ln 24}{\ln 5} \approx 1.9746$$

6. (5pts) Solve the equation:

$$\log_6(9-x) + \log_6(x+3) = 2$$

$$\log_6((9-x)(x+3)) = 2$$

$$6^2 = (9-x)(x+3)$$

$$36 = 9x - x^2 + 27 - 3x$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3$$

$$\text{Test: } \log_6(9-3) + \log_6(3+3) \stackrel{?}{=} 2$$

$$1 + 1 = 2$$

7. (6pts) In 1999, Spurt City was recorded to have 10,000 residents. In 2007, records show it had 18,000 residents. Do the following, assuming the growth was exponential.

a) Find the growth rate. Write the function that expresses the population  $t$  years from 1999.

b) Find the population in 2005.

a)	$t$	$N$
1999	0	10
2007	8	18

$$P = 10e^{kt}$$

$$k = ?$$

$$18 = 10e^{k \cdot 8} \quad | \div 10$$

$$1.8 = e^{k \cdot 8} \quad | \ln$$

$$\ln 1.8 = k \cdot 8 \quad | \div 8$$

$$k = \frac{\ln 1.8}{8} \approx 0.07347 \dots$$

$$P(t) = 10e^{0.07347t}$$

$$\begin{aligned} \text{b) } P(6) &= 10e^{0.07347 \cdot 6} \\ &= 15.5401 \end{aligned}$$

15,540 residents in 2005.