

1. (8pts) Put the following expressions into standard form $a + bi$:

$$\begin{aligned} \text{a) } (3 - 4i)[2 + i(1 + 5i)] &= (3 - 4i)(2 + i + \underbrace{5i^2}_{-5}) = (3 - 4i)(-3 + i) \\ &= -9 + 12i + 3i - \underbrace{4i^2}_{-4} = -5 + 15i \end{aligned}$$

$$\text{b) } \frac{7 - 3i}{4 + i} = \frac{7 - 3i}{4 + i} \cdot \frac{4 - i}{4 - i} = \frac{28 - 12i - 7i + 3i^2}{4^2 - i^2} = \frac{25 - 19i}{16 - (-1)} = \frac{25 - 19i}{17} = \frac{25}{17} - \frac{19}{17}i$$

$$\text{c) (justify also) } i^{99} = i^3 = i^2 \cdot i = -i$$

$$\frac{99}{4} = 24, \text{ rem } 3$$

Solve the equations algebraically:

$$\text{2. (5pts) } x^4 + 9x^2 - 22 = 0 \quad u = -11, 2$$

$$\text{Set } u = x^2$$

$$u^2 + 9u - 22 = 0$$

$$(u + 11)(u - 2) = 0$$

$$x^2 = -11$$

$$x = \pm \sqrt{-11}$$

$$= \pm \sqrt{11}i$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

$$\text{3. (5pts) } x + 3 = \sqrt{2x + 41} \quad |^2$$

$$x = -8 \text{ or } 4$$

$$x^2 + 6x + 9 = 2x + 41 \quad | -2x - 41$$

$$x^2 + 4x - 32 = 0$$

$$(x + 8)(x - 4) = 0$$

test in original equation:

$$-8 + 3 \stackrel{?}{=} \sqrt{-16 + 41}$$

$$-5 \stackrel{?}{=} \sqrt{25} \text{ no}$$

$$4 + 3 \stackrel{?}{=} \sqrt{8 + 41}$$

$$7 = \sqrt{49} \text{ yes}$$

only $x = 4$

is the solution.

4. (6pts) How much pure water should be added to 30ml of a 30% solution of muriatic acid to get a 12% solution?

$$\boxed{\begin{array}{c} x \text{ ml} \\ \text{water} \end{array}} + \boxed{\begin{array}{c} 30 \text{ ml} \\ 30\% \text{ acid} \end{array}} = \boxed{\begin{array}{c} x+30 \\ 12\% \text{ sol.} \end{array}}$$

$$0 + 0.3 \cdot 30 = 0.12(x+30)$$

$$9 = 0.12x + 3.6$$

$$5.4 = 0.12x$$

$$x = \frac{5.4}{0.12} = 45 \text{ ml.}$$

5. (6pts) A bathroom tub fills in 15 minutes if the faucet is open and the stopper is in place. With the faucet closed and the stopper removed, the tub empties in ~~15~~²⁰ minutes. How long does it take to fill the tub if the faucet is open and the stopper is removed? (Hint: think of faucet and stopper as people contributing to or taking away from a job.)

Let T = time to fill tub with faucet open and stopper open (in minutes)

$\frac{1}{T}$ = portion of tub that fills in 1 min under conditions above

$\frac{1}{15}$ = portion of tub that fills in 1 min with stopper closed

$\frac{1}{20}$ = portion of tub that drains in 1 min with faucet closed

$$\frac{1}{T} = \frac{1}{15} - \frac{1}{20}$$

$$\frac{1}{T} = \frac{1}{60}$$

$$T = 60 \text{ min.}$$

$$\frac{1}{T} = \frac{4-3}{60}$$