1. (6pts) Solve for x:

$a^2x + b = b^2x - a$

2. (8pts) Solve the inequality and write the solution in interval notation: $|x - 7| \geq 2$.

3. (12pts) Find the equation of the line that contains $(2, 4)$ and is perpendicular to the line through $A = (-1, 3)$ and $B = (5, -2)$. Sketch both lines on the same coordinate system.

4. (12pts) Put the complex number into form $a + bi$.

a) $\dfrac{(1 + i)(3 + 2i)}{i} =$

b) (explain) $i^{31} =$

5. (10pts) Let $A = (-1, 4)$, $B = (-3, 8)$.
a) Find the midpoint M of the segment AB.
b) Verify that the distances from M to A and B are equal.

6. (14pts) The equation $y = -x^4 + 6x^2 - 4x + 5$ is given.
a) Use your calculator to accurately sketch the graph of the equation on paper. Indicate your viewing window.
b) What is the y-intercept of the graph?
c) Using your calculator, solve the inequality $-x^4 + 6x^2 - 4x + 5 > 0$. Write your answer in interval notation with accuracy 4 decimal points.

Solve the equations, using complex numbers if necessary:

7. (8pts) $x^2 - 7x + 5 = 0$

8. (10pts) $x^4 - 3x^2 - 40 = 0$

9. (8pts) Find the equation of the circle whose center is $(3, 4)$ and contains the point $(4, -1)$. Sketch the circle. *(Hint: what is the radius?)*

10. (12pts) How many liters of water needs to be added to 4 liters of a 15% solution of hydrochloric acid in order to get a 10% solution? Don't forget to write down what your variable means.

Bonus (10pts) Farmer Jeremiah has 700ft of fencing. Using all of this fencing, he plans to enclose a rectangular plot of land and divide it into two triangles by stringing fence along one diagonal of the rectangular plot. If the length of the rectangular region has to be 50ft more than the width, what are the dimensions of the enclosure?

**1.** (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.
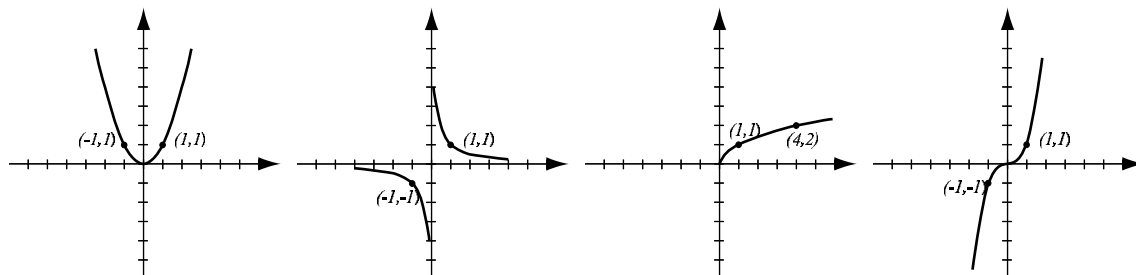
2. (8pts) Let $f(x) = \dfrac{x^2 - x - 6}{x^2 - 3x - 28}$.

a) Find the domain of this function.
b) Where does the graph of this rational function have vertical asymptotes?

3. (12pts) Use the graph of the function f
at right to answer the following questions.
a) What are the x-intercepts?
b) What is the range?
c) Where is the function increasing?
d) Where does f have a local maximum?
What is its value?
e) For which x's is $f(x) \le 0$?
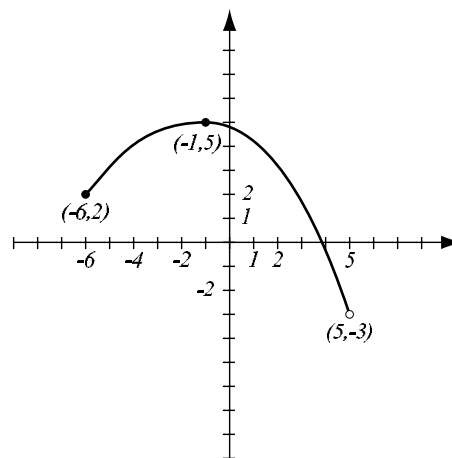
4. (12pts) "Everything but", a small manufacturer of kitchen sinks, produced 20 sinks in one day at a cost of \$3100. On another day, "Everything but" produced 32 sinks at a cost of \$3640.

a) Write the cost as a function of number x of sinks produced, assuming it is linear.
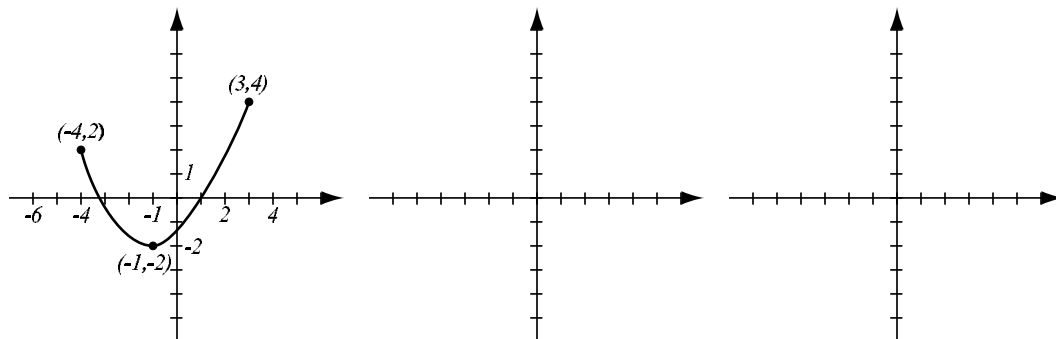
b) What is the cost of producing 29 sinks?

5. (11pts) Let $f(x) = 5 - x$ and $g(x) = x^2 - 1$. Find the following and simplify where possible:

$\frac{f}{g}(2) =$

$g(3t + 2) =$

$f(4 - u^2) =$

6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x - 2)$ and $1.5f(x)$ and label all the relevant points.

7. (12pts) The quadratic function $f(x) = x^2 - x - 7$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

**8.** (17pts) Consider the polynomial $f(x) = x^3 - 25x$.

a) Algebraically determine whether this function is even, odd or neither.

b) Algebraically find the x-intercepts of the graph and the y-intercept.

c) Sketch the graph of the function on paper (large, clear, with units — make Dad proud!).

d) Does your graph support the conclusion in a)? Explain why.

e) Use your calculator to determine where the function is decreasing. (Accuracy: 4 decimal points.)

9. (10pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 3x - 2, & \text{if } -4 < x < 2 \\ 4 - x, & \text{if } 2 \le x. \end{cases}$$

Bonus (10pts) A square piece of cardboard is 10 centimeters wide. To make a box with an open top, a square piece whose side is x is cut from each corner and the remaining flaps are folded upwards.
a) Express the volume V of the box as a function of x. What is the domain of this function?
b) Use your calculator to draw a rough graph of the function $V(x)$.
c) Use your calculator to find the size x of the piece that has to be removed in order to get a box of the greatest volume. What is the maximal volume?
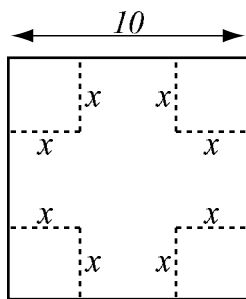
1. (10pts) Let $f(x) = x^2 - \sqrt{x} + 3$ and $g(x) = 3x + 5$. Find $(f \circ g)(x)$ and $(g \circ g)(x)$ and simplify.

2. (12pts) The graph of f is given.
a) Explain why f has an inverse.
b) Find the graph of its inverse function.
Label the relevant points.
c) Find the domain and range of both f and f^{-1}.
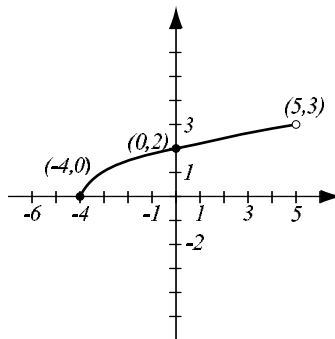
3. (8pts) Evaluate without using the calculator:

$\log_7 49 =$ $\log_8 2 =$ $\log_b \sqrt[7]{b^4} =$ $\log_a \frac{1}{a^2} =$

4. (14pts) Solve the equations

$$5^{2x-3} = e$$

$$\log_3(x+6) + \log_3 x = 3$$

5. (6pts) Write as a sum of logarithms. Express powers as factors. Simplify if possible.

$$\ln\left(e^2 \cdot \sqrt{5x+2}\right) =$$

6. (8pts) Write as a single logarithm (get that 2 inside the single logarithm somehow!) and simplify.

$$6\log_5(x-1)^4 - 8\log_5 \sqrt[4]{(x-1)^3} - 2 =$$

7. (8pts) Find the inverse of $f(x) = 4e^{x+3}$.

8. (14pts) Iodine 131, which decays according to the exponential law and whose half-life is 8 days, was released into the atmosphere as a result of the accident at the nuclear power plant Chernobyl in 1986. This contaminated hay in many parts of Europe.

a) Find the k from the exponential law.

b) If it is all right to feed the hay to cows when 10% of iodine 131 remains, how long do farmers have to wait to use this hay?

c) Without much computation, how long do you have to wait for iodine 131 to decay to $\frac{1}{8}$ of the original amount?

9. (4pts) In a coordinate system, roughly draw angles of measure $-150°$ and $\frac{3\pi}{5}$ radians.

10. (4pts) Indicate both the radian and degree measure under the following angles. (Use equally-spaced auxiliary lines to help you determine what the angles are.)
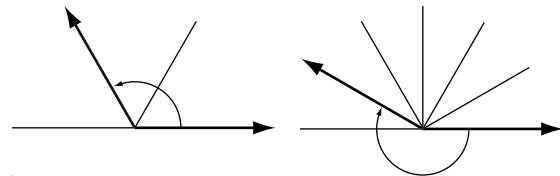
11. (8pts) The city of Veracruz, Mexico is directly south of Dallas, Texas. If the latitudes of Veracruz and Dallas are approximately $19°$ and $32.5°$, respectively, what is the distance between the two cities? Assume radius of Earth is 3960 miles.

12. (4pts) A wheel is rotating with angular speed 48π radians per minute. How many rounds per minute is this?

Bonus (10pts) Show that $\log_a \left(x + \sqrt{x^2 - 1}\right) + \log_a \left(x - \sqrt{x^2 - 1}\right) = 0$.

1. (16pts) Without using the calculator, find the exact values of the following expressions. Draw the unit circle and the appropriate angle under the expression.
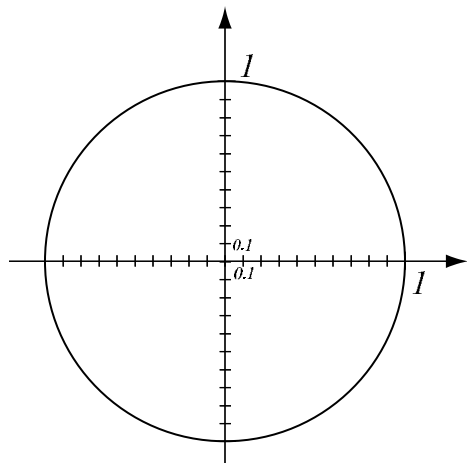
$$\cos\frac{3\pi}{4} = \qquad\qquad \tan\frac{4\pi}{3} = \qquad\qquad \arccos\frac{\sqrt{3}}{2} = \qquad\qquad \arcsin(-1) =$$

2. (8pts) Use the picture below to estimate $\sin 70°$ and $\arccos(-0.6)$ (in degrees). Then evaluate these numbers using a calculator and compare your answers.

3. (10pts) Draw two periods of the graph of $y = 2\cos(2\pi x)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).

4. (10pts) Find all the solutions of the equation $\dfrac{\tan(3\theta)}{\sqrt{3}} - 1 = 0$.

5. (10pts) Find the exact value of $\sin(\arctan(-5))$. Draw a picture and do not use the calculator.

6. (10pts) Find the exact values of the expressions below. Draw a picture if helpful and do not use the calculator. If either expression is undefined, explain why.

$\sin(\arcsin 3) =$

$\arcsin\left(\sin \dfrac{9\pi}{7}\right) =$

7. (8pts) A 20-ft ladder leans against the wall and makes an angle of 78° degrees with the floor. How far from the base of the wall is the bottom of the ladder?

8. (16pts) You take a sighting of an object at the top of a building from a certain point and find that the angle of elevation is 63°. Then you move in a straight line towards the building, pass through the building, exit, and stop some distance away on the other side. Looking back, you see the same object at the top of the building at angle of elevation 72°. If the distance between the points where you took the sightings is 110 meters, how tall is the building (accuracy: 4 decimal points)?

9. (12pts) Solve the triangle: $a = 4$, $b = 7$, $c = 5$ (accuracy: 4 decimal points).

Bonus. (10pts) Among all isosceles triangles (two sides have equal length) whose legs are 10in, find the one with the biggest area by following these directions:
a) Draw three different triangles showing that you get a different areas depending on the angle θ between the legs (no computation needed here, just the pictures).
b) Write the formula for the area of the triangle as a function of θ.
c) Graph the function $A(\theta)$ and find its maximum (use degrees for θ). What interval should you consider for θ?

1. (8pts) Find the equation of the line that that is parallel to the line $5x - 2y = 10$ and passes through the point $(3, -1)$. Sketch both lines in a coordinate system.

2. (4pts) Solve the inequality $3 - 2x \geq 4$ and write your answer using interval notation.

3. (12pts) The quadratic function $f(x) = x^2 - 4x + 1$ is given. Do the following without using the calculator.
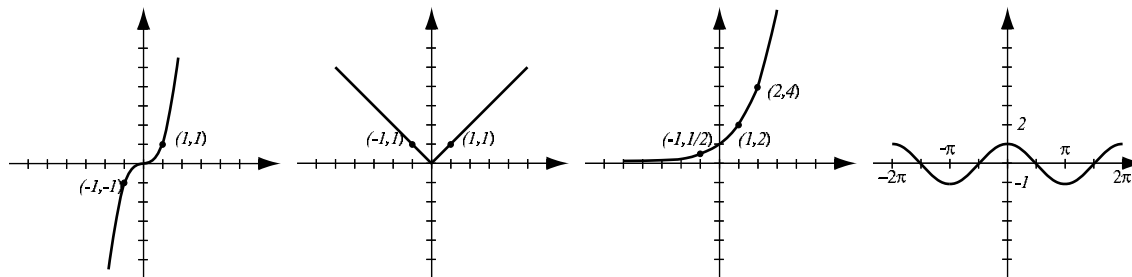a) Find the x-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

4. (10pts) Solve the equation: $x - 2 = \sqrt{8 - x}$.

5. (8pts) The following are graphs of basic functions that we have had in this course. Write the equation of the graph under each one.
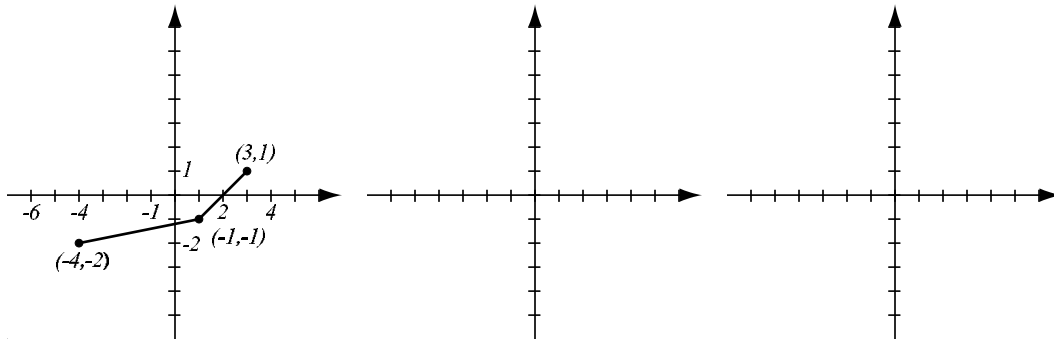

6. (9pts) Use the graph of the function f at right to answer the following questions.
a) What is $f(0)$?
b) What are the x-intercepts?
c) State the intervals of x's where $f(x) < 0$.
d) What are the solutions of the equation $f(x) = -3$?
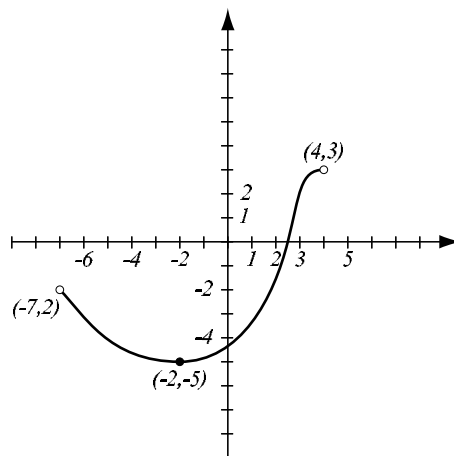
7. (8pts) The graph of $f(x)$ is drawn below. Find the graphs $f(x) + 3$ and $-3f(x)$ and label all the relevant points.

8. (16pts) The polynomial $f(x) = x^4 - 2x^3 + 5x^2 - 8$ is given. Use your calculator to solve the following with accuracy 4 decimal points.
a) Find the x-intercepts and the y-intercept.
b) Find the intervals of increase and decrease of this function.
c) What is the range of f?
d) Algebraically determine whether this function is even, odd or neither.
e) Sketch the graph of the function on paper (large and clear — make Grampa proud!).
f) Does your graph support the conclusion in d)? Explain why.

9. (6pts) Let $f(x) = x - 7$, $g(x) = \dfrac{2x - 1}{4x + 3}$ Find $(g \circ f)(x)$ and simplify.

10. (6pts) Evaluate without using the calculator:

$\log_4 64 =$ $\qquad\qquad$ $\log_5 \frac{1}{25} =$ $\qquad\qquad$ $\log_{36} 6 =$

11. (6pts) Write as a sum of logarithms. Express powers as factors. Simplify if possible.

$\log_7 \left(49(x + 3)^5 \cdot \sqrt{(x - 7)^3} \right) =$

12. (8pts) Solve the equation: $5^{x+2} = 7^x$.

13. (9pts) Without using the calculator, find the exact values of the following expressions. Draw the unit circle and the appropriate angle under the expression.

$$\cos 240° = \qquad\qquad \tan \frac{2\pi}{3} = \qquad\qquad \arcsin \frac{1}{2} =$$

14. (6pts) Find the exact value of the expression below. Draw a picture and do not use the calculator.

$$\arcsin \left(\sin \frac{9\pi}{7} \right) =$$

15. (12pts) Solve the triangle if $a = 4$, $b = 7$ and $\beta = 27°$

16. (12pts) How many liters of a 10% solution of muriatic acid needs to be added to 3 liters of a 45% solution of muriatic acid in order to get a 15% solution?

Bonus (14pts) A gardener has a piece of fencing 20 feet long that she wants to use to enclose a triangular plot between two walls (see the picture). She can position the fencing so that angle θ is anything between $0°$ and $90°$ and wishes to maximize the enclosed area.
a) Draw the position of the fencing for angles $\theta = 45°$ and $\theta = 30°$, and find the areas of the resulting triangles (exact values here, not decimal approximations).
b) Write the formula for the area of the triangle $A(\theta)$ as a function of θ.
c) Graph the function $A(\theta)$ and find its maximum (use degrees for θ).