Fall '07/MAT 150/Final Exam **Name:** Solution *Show all your work.*

1. (8pts) Find the equation of the line that that is parallel to the line $5x - 2y = 10$ and passes through the point $(3, -1)$. Sketch both lines in a coordinate system.

$5x - 2y = 10$

$2y = 5x - 10 \quad / \div 2$

$y = \frac{5}{2}x - 5$

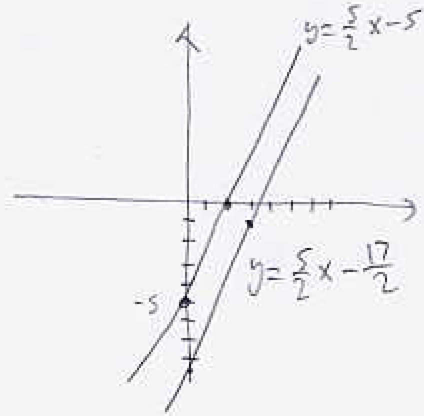
slope $= \frac{5}{2}$

Desired line:

$y - (-1) = \frac{5}{2}(x - 3)$

$y = \frac{5}{2}x - \frac{15}{2} - 1$

$y = \frac{5}{2}x - \frac{17}{2}$



2. (4pts) Solve the inequality $3 - 2x \geq 4$ and write your answer using interval notation.

$3 - 2x \geq 4 \quad / -3$

$-2x \geq 1 \quad / \div -2$

$x \leq -\frac{1}{2}$

x is in $\left(-\infty, -\frac{1}{2}\right]$

3. (12pts) The quadratic function $f(x) = x^2 - 4x + 1$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) $x^2 - 4x + 1 = 0$

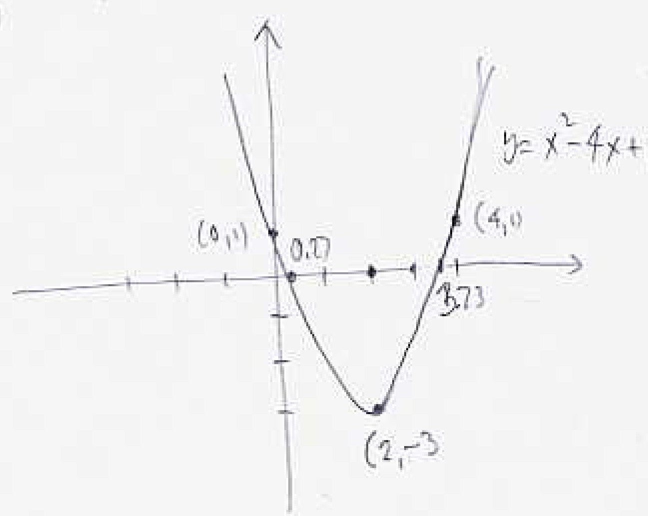
$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{4 \pm \sqrt{12}}{2}$

$= \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3} \approx \begin{matrix} 3.73 \\ 0.27 \end{matrix}$

b) $x = -\frac{(-4)}{2 \cdot 1} = 2$

$y = 2^2 - 4 \cdot 2 + 1 = -3$

c)



4. (10pts) Solve the equation: $x - 2 = \sqrt{8 - x}$.

$x-2=\sqrt{8-x}\ \Big|^2$

$x^2-4x+4=8-x$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4,-1$

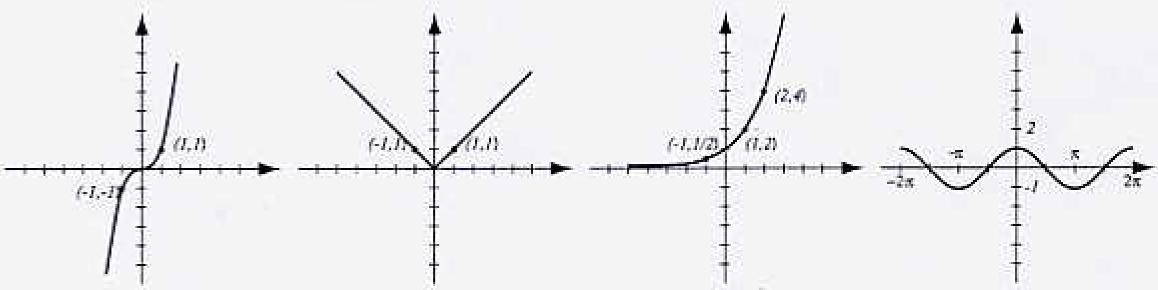
Test answer: $4-2 \overset{?}{=} \sqrt{8-4}$ yes

$-1-2 \overset{?}{=} \sqrt{8-(-1)}$ no

$-3 \neq 3$

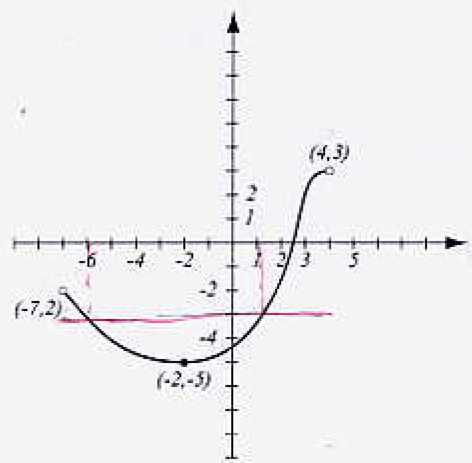
Solution is only $x=4$

5. (8pts) The following are graphs of basic functions that we have had in this course. Write the equation of the graph under each one.



$y=x^3$ $\quad$ $y=|x|$ $\quad$ $y=2^x$ $\quad$ $y=\cos x$

6. (9pts) Use the graph of the function f at right to answer the following questions.
a) What is $f(0)$?
b) What are the x-intercepts?
c) State the intervals of x's where $f(x) < 0$.
d) What are the solutions of the equation $f(x) = -3$?



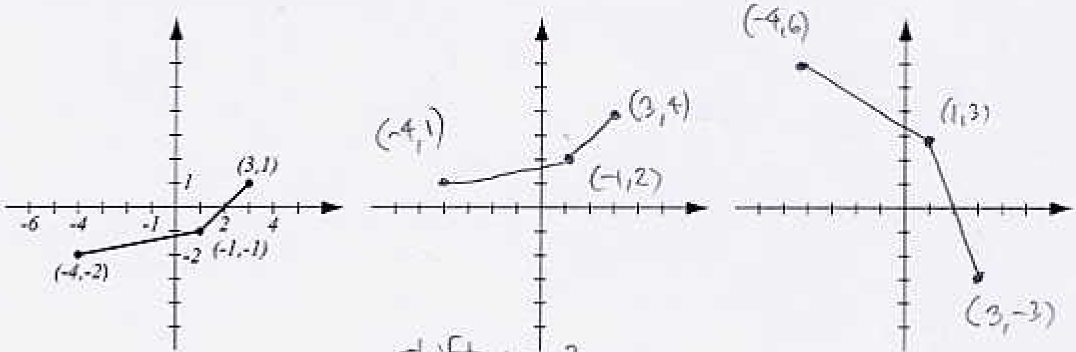
a) $f(0) \approx -4.4$

b) $x \approx 2.5$

c) on $(-7, 2.5)$

d) $x=-6$ or $x=1.2$

7. (8pts) The graph of $f(x)$ is drawn below. Find the graphs $f(x)+3$ and $-3f(x)$ and label all the relevant points.



stretch vertically by factor 3, reflect in x-axis

shift up 3

8. (16pts) The polynomial $f(x) = x^4 - 2x^3 + 5x^2 - 8$ is given. Use your calculator to solve the following with accuracy 4 decimal points.
a) Find the x-intercepts and the y-intercept.
b) Find the intervals of increase and decrease of this function.
c) What is the range of f?
d) Algebraically determine whether this function is even, odd or neither.
e) Sketch the graph of the function on paper (large and clear — make Grampa proud!).
f) Does your graph support the conclusion in d)? Explain why.

a) y-int $= -8$
x-int: -1, 1.3883

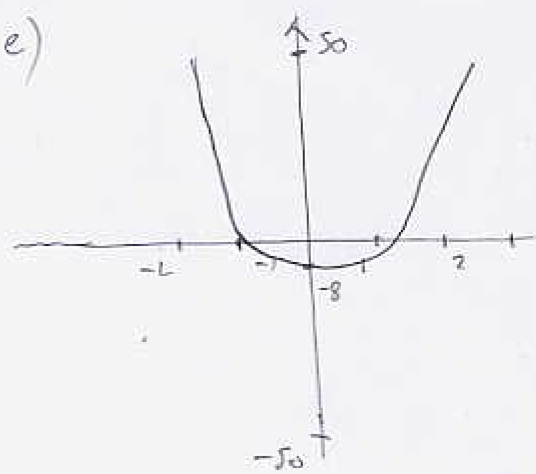
b) local min is at 0
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$

c) Range is $[-8, \infty)$

d) $f(-x) = (-x)^4 - 2(-x)^3 + 5(-x)^2 - 8$
$= x^4 + 2x^3 + 5x^2 - 8 \neq f(x)$
$\neq -f(x)$

It is neither

e)



f) The graph is neither symmetric wrt y-axis nor wrt the origin, so the function is neither odd nor even.

9. (6pts) Let $f(x) = x - 7$, $g(x) = \dfrac{2x-1}{4x+3}$ Find $(g \circ f)(x)$ and simplify.

$$g(f(x)) = g(x-7) = \frac{2(x-7)-1}{4(x-7)+3} = \frac{2x-14-1}{4x-28+3} = \frac{2x-15}{4x-25}$$

10. (6pts) Evaluate without using the calculator:

$\log_4 64 = 3$ $\qquad$ $\log_5 \frac{1}{25} = -2$ $\qquad$ $\log_{36} 6 = \frac{1}{2}$

$4^{?} = 64$ $\qquad$ $5^{?} = \frac{1}{25}$ $\qquad$ $36^{?} = 6$

$\sqrt{36} = 6$

11. (6pts) Write as a sum of logarithms. Express powers as factors. Simplify if possible.

$$\log_7\left(49(x+3)^5 \cdot \sqrt{(x-7)^3}\right) = \log_7 49 + \log_7 (x+3)^5 + \log_7 (x-7)^{3/2}$$

$$= 2 + 5\log_7(x+3) + \frac{3}{2}\log_7(x-7)$$

12. (8pts) Solve the equation: $5^{x+2} = 7^x$.

$5^{x+2} = 7^x \quad | \ln$

$\ln 5^{x+2} = \ln 7^x$

$(x+2)\ln 5 = x \ln 7$

$x\ln 5 + 2\ln 5 = x\ln 7$

$x\ln 5 - x\ln 7 = -2\ln 5$

$x(\ln 5 - \ln 7) = -2\ln 5$

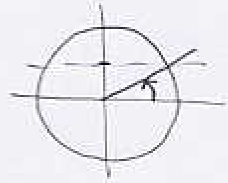
$x = \dfrac{-2\ln 5}{\ln 5 - \ln 7} = \dfrac{2\ln 5}{\ln 7 - \ln 5}$

≈ 9.5665

13. (9pts) Without using the calculator, find the exact values of the following expressions. Draw the unit circle and the appropriate angle under the expression.

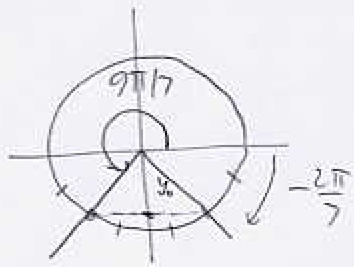
$\cos 240^\circ = -\frac{1}{2}$ $\qquad$ $\tan \frac{2\pi}{3} = -\sqrt{3}$ $\qquad$ $\arcsin \frac{1}{2} = \frac{\pi}{6}$

$180+60^\circ$

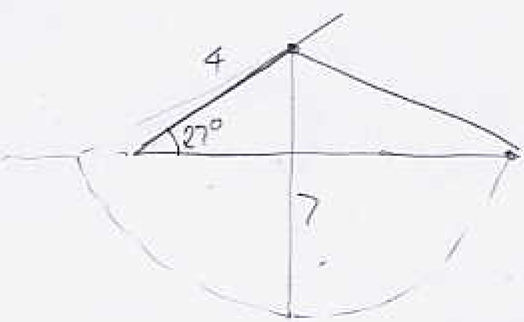


14. (6pts) Find the exact value of the expression below. Draw a picture and do not use the calculator.

$\arcsin \left(\underbrace{\sin \frac{9\pi}{7}}_{y_0} \right) = \arcsin y_0 = -\frac{2\pi}{7}$



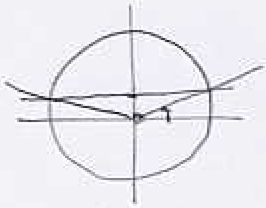
15. (12pts) Solve the triangle if $a = 4$, $b = 7$ and $\beta = 27^\circ$



Looks like one solution

$\frac{\sin \alpha}{4} = \frac{\sin 27^\circ}{7}$

$\sin \alpha = \frac{4 \sin 27^\circ}{7} \approx 0.2594$



$\alpha = \arcsin 0.2594$ or $180^\circ - \arcsin 0.2594$

$= 15.0358$ $\qquad$ $\alpha = 164.9642$

$\gamma = 180^\circ - (27^\circ + 15.0358^\circ)$ $\qquad$ $\gamma = 180 - (27 + 164.9642)$

$= 137.9642^\circ$ $\qquad$ $= -11.9642$

not possible

$\frac{\sin 27^\circ}{7} = \frac{\sin 137.9642^\circ}{c}$

$c = \frac{7 \sin 137.9642}{\sin 27^\circ} \approx 10.3244$

16. (12pts) How many liters of a 10% solution of muriatic acid needs to be added to 3 liters of a 45% solution of ~~hydrochloric~~ muriatic acid in order to get a 15% solution?

x = liters of muriatic acid

$$\boxed{\begin{matrix} x \\ 10\%\,\text{sol} \end{matrix}} + \boxed{\begin{matrix} 3\ell \\ 45\%\,\text{sol} \end{matrix}} = \boxed{\begin{matrix} x+3 \\ 15\% \end{matrix}}$$

$$0.1x + 0.45 \cdot 3 = 0.15(x+3)$$

$$0.1x + 1.35 = 0.15x + 0.45$$

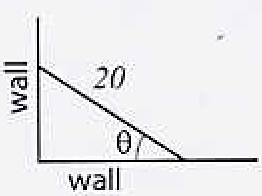
$$1.35 - 0.45 = 0.15x - 0.1x$$

$$0.9 = 0.05x$$

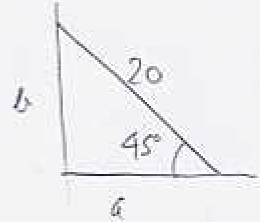
$$x = \frac{0.9}{0.05} = 18$$

18 liters should be added.

Bonus (14pts) A gardener has a piece of fencing 20 feet long that she wants to use to enclose a triangular plot between two walls (see the picture). She can position the fencing so that angle θ is anything between 0° and 90° and wishes to maximize the enclosed area.
a) Draw the position of the fencing for angles $\theta = 45°$ and $\theta = 30°$, and find the areas of the resulting triangles (exact values here, not decimal approximations).
b) Write the formula for the area of the triangle $A(\theta)$ as a function of θ.
c) Graph the function $A(\theta)$ and find its maximum (use degrees for θ).



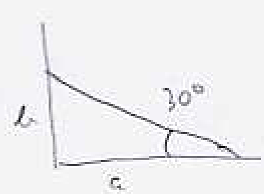
a)



$\frac{b}{20} = \sin 45° = \frac{\sqrt{2}}{2} \Rightarrow b = 10\sqrt{2}$

$\frac{a}{20} = \cos 45° = \frac{\sqrt{2}}{2} \Rightarrow a = 10\sqrt{2}$

$A = \frac{1}{2} a \cdot b = \frac{1}{2} \cdot 10\sqrt{2} \cdot 10\sqrt{2} = 100$

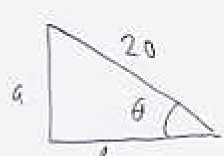


$a = 20 \cos 30° = 20 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3}$

$b = 20 \sin 30° = 20 \cdot \frac{1}{2} = 10$

$A = \frac{1}{2} ab = \frac{1}{2} 10\sqrt{3} \cdot 10 = 50\sqrt{3} \approx 86.6025$

b)



$\frac{a}{20} = \sin\theta$

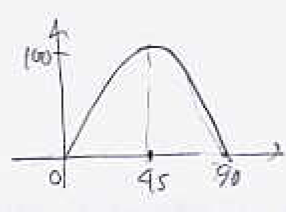
$\frac{b}{20} = \cos\theta$

$a = 20\sin\theta, \quad b = 20\cos\theta$

$A = \frac{1}{2} ab = \frac{1}{2} 20\sin\theta \cdot 20\cos\theta$

$= 200 \sin\theta \cos\theta$

c)



Max occurs for $\theta = 45°$

Max area is 100 ft^2