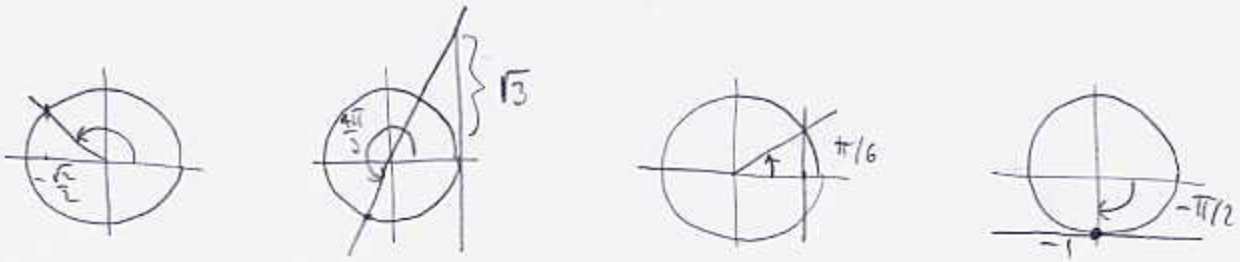
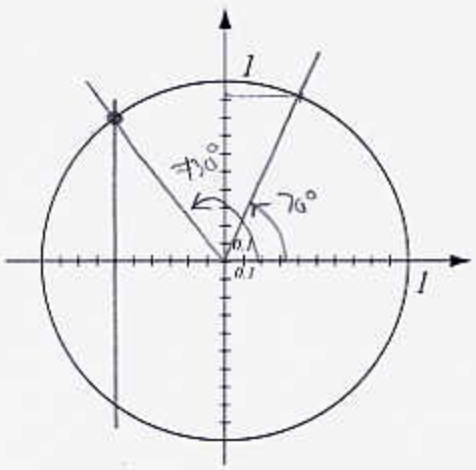
Fall '07/MAT 150/Exam 4 **Name:** Solution *Show all your work.*

1. (16pts) Without using the calculator, find the exact values of the following expressions. Draw the unit circle and the appropriate angle under the expression.

$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ $\tan\frac{4\pi}{3} = \sqrt{3}$ $\arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6}$ $\arcsin(-1) = -\frac{\pi}{2}$



2. (8pts) Use the picture below to estimate $\sin 70°$ and $\arccos(-0.6)$ (in degrees). Then evaluate these numbers using a calculator and compare your answers.

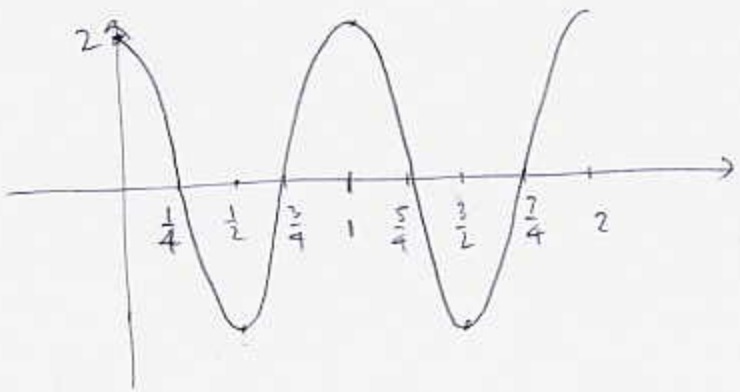


	est	calc
$\sin 70° \approx$	0.92	0.93
$\arccos(-0.6) \approx$	130°	126.87°

3. (10pts) Draw two periods of the graph of $y = 2\sin(2\pi x)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).

Amplitude $= 2$

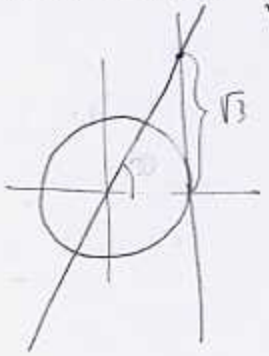
period $= \frac{2\pi}{2\pi} = 1$



4. (10pts) Find all the solutions of the equation $\frac{\tan(3\theta)}{\sqrt{3}} - 1 = 0$.

$\frac{\tan(3\theta)}{\sqrt{3}} = 1$

$\tan(3\theta) = \sqrt{3}$



$3\theta = \frac{\pi}{3} + k\pi$

$\theta = \frac{\pi}{9} + k \cdot \frac{\pi}{3}$

5. (10pts) Find the exact value of sin(arctan 5). Draw a picture and do not use the calculator.

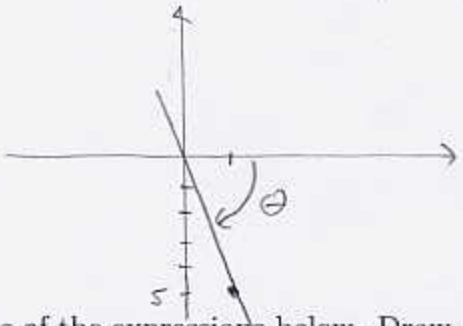
$\sin(\arctan 5) = \sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{26}}$

$r^2 = (-5)^2 + 1^2$

$r^2 = 26$

$\tan\theta = -5 = \frac{y}{x} = \frac{-5}{1}$

θ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

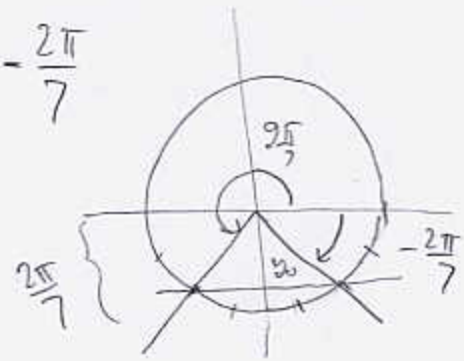


6. (10pts) Find the exact values of the expressions below. Draw a picture if helpful and do not use the calculator. If either expression is undefined, explain why.

$\sin(\arcsin 3) =$ not defined since $\arcsin x$ is defined only for $-1 \le x \le 1$

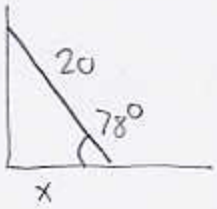
$\arcsin\left(\sin\frac{9\pi}{7}\right) = \arcsin y_0 = -\frac{2\pi}{7}$

not in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$\frac{2\pi}{7} = 1\frac{2}{7}\pi$

7. (8pts) A 20-ft ladder leans against the wall and makes an angle of 78° degrees with the floor. How far from the base of the wall is the bottom of the ladder?

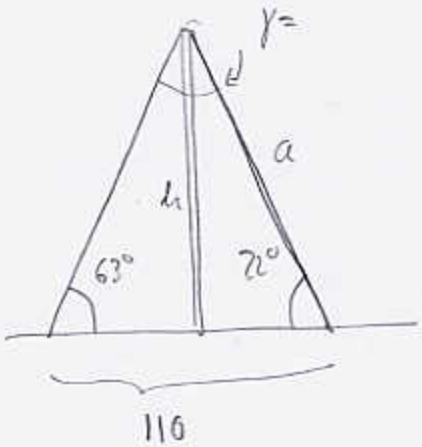


$$\frac{x}{20} = \cos 78^\circ$$

$$x = 20 \cos 78^\circ$$

$$= 4.1582$$

8. (16pts) You take a sighting of an object at the top of a building from a certain point and find that the angle of elevation is 63°. Then you move in a straight line towards the building, pass through the building, exit, and stop some distance away on the other side. Looking back, you see the same object at the top of the building at angle of elevation 72°. If the distance between the points where you took the sightings is 110 meters, how tall is the building (accuracy: 4 decimal points)?



$h = ?$

Find a first

$$\frac{\sin 63^\circ}{a} = \frac{\sin 45^\circ}{110}$$

$$a = \frac{110 \sin 63^\circ}{\sin 45^\circ} = 138.6081$$

$y = 180^\circ - (63^\circ + 72^\circ)$

$= 180^\circ - 135^\circ$

$= 45^\circ$

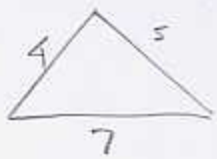
Now: $\frac{h}{a} = \sin 72^\circ$

$h = a \sin 72^\circ = 138.6081 \cdot \sin 72^\circ$

$= 131.8241$ m

9. (12pts) Solve the triangle: $a = 4$, $b = 7$, $c = 5$ (accuracy: 4 decimal points).

$$\cos\gamma = \frac{a^2+b^2-c^2}{2ab} = \frac{4^2+7^2-5^2}{2\cdot 4\cdot 7} = \frac{40}{56} = \frac{5}{7}$$



$$\cos\beta = \frac{a^2+c^2-b^2}{2ac} = \frac{4^2+5^2-7^2}{2\cdot 4\cdot 5} = -\frac{8}{40} = -\frac{1}{5}$$

$\gamma = \arccos\frac{5}{7} \approx 44.4153^\circ$ $\quad \beta = \arccos\left(-\frac{1}{5}\right) = 101.5370$

$\alpha = 180^\circ - (44.4153^\circ + 101.5370^\circ)$

$= 34.0477$

Bonus. (10pts) Among all isosceles triangles (two sides have equal length) whose legs are 10in, find the one with the biggest area by following these directions:
a) Draw three different triangles showing that you get a different areas depending on the angle θ between the legs (no computation needed here, just the pictures).
b) Write the formula for the area of the triangle as a function of θ.
c) Graph the function $A(\theta)$ and find its maximum (use degrees for θ). What interval should you consider for θ?

a)

b)

$A = \frac{1}{2} b\cdot h = \frac{1}{2}\cdot 10\sin\frac{\theta}{2}\cdot 10\cos\frac{\theta}{2}$

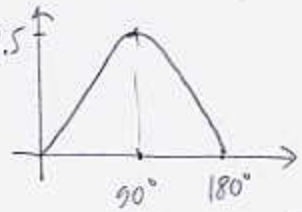
$A(\theta) = 25\sin\frac{\theta}{2}\cos\frac{\theta}{2} = (12.5\sin\theta)$

$\frac{h}{10} = \cos\frac{\theta}{2}$ $\quad h = 10\cos\frac{\theta}{2}$

$\frac{\frac{b}{2}}{10} = \sin\frac{\theta}{2}$

$b = 20\sin\frac{\theta}{2}$

c) $0 \le \theta \le 180^\circ$



Max occurs for $\theta = 90^\circ$

Max area is $12.5\ in^2$