Fall '07/MAT 150/Exam 3 **Name:** Solution *Show all your work.*

1. (10pts) Let $f(x) = x^2 - \sqrt{x} + 3$ and $g(x) = 3x + 5$. Find $(f \circ g)(x)$ and $(g \circ g)(x)$ and simplify.

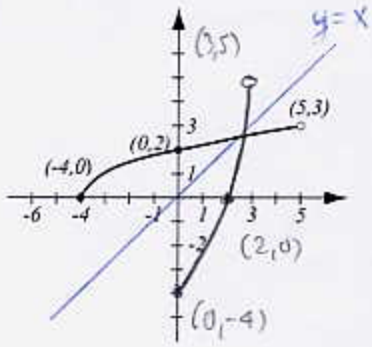
$(f \circ g)(x) = f(g(x)) = f(3x+5) = (3x+5)^2 - \sqrt{3x+5} + 3$

$= 9x^2 + 30x + 28 - \sqrt{3x+5}$

$(g \circ g)(x) = g(g(x)) = g(3x+5) = 3(3x+5)$

$= 9x + 15$

2. (12pts) The graph of f is given.
a) Explain why f has an inverse.
b) Find the graph of its inverse function. Label the relevant points.
c) Find the domain and range of both f and f^{-1}.



a) because it passes the horizontal line-test

c)

	f	f^{-1}
domain	$[-4,5)$	$[0,3)$
range	$[0,3)$	$[-4,5)$

3. (8pts) Evaluate without using the calculator:

$\log_7 49 = 2$ $\quad 7^? = 49$

$\log_8 2 = \frac{1}{3}$ $\quad 8^? = 2$ $\quad \sqrt[3]{8} = 2$ $\quad 8^{1/3} = 2$

$\log_b \sqrt[7]{b^4} = \frac{4}{7}$ $\quad b^? = b^{\frac{4}{7}}$

$\log_a \frac{1}{a^2} = -2$ $\quad a^? = \frac{1}{a^2} = a^{-2}$

4. (14pts) Solve the equations

$5^{2x-3} = e$ | ln

$\ln 5^{2x-3} = 1$

$(2x-3)\ln 5 = 1$

$2x-3 = \frac{1}{\ln 5}$

$2x = 3 + \frac{1}{\ln 5}$

$x = \frac{3+\frac{1}{\ln 5}}{2} \approx 1.8107$

$\log_3(x+6) + \log_3 x = 3$

$\log_3 (x+6)x = 3$

$(x+6)x = 3^3$

$x^2+6x-27 = 0$

$(x+9)(x-3) = 0$

$x = -9, 3$

$\boxed{x=3}$

test:

$\log_3(-9+6) + \log_3 -9$

↑ not defined ↑

$\log_3(3+6) + \log_3 3 \stackrel{?}{=} 3$

$2 + 1 = 3$ yes

5. (6pts) Write as a sum of logarithms. Express powers as factors. Simplify if possible.

$\ln\left(e^2 \cdot \sqrt{5x+2}\right) = \ln e^2 + \ln (5x+2)^{\frac{1}{2}}$

$= 2 + \frac{1}{2}\ln(5x+2)$

6. (8pts) Write as a single logarithm (get that 2 inside the single logarithm somehow!) and simplify.

$6\log_5(x-1)^4 - 8\log_5 \sqrt[4]{(x-1)^3} - 2 = \log_5((x-1)^4)^6 - \log_5\left((x-1)^{\frac{3}{4}}\right)^8 - \log_5 25$

$= \log_5 (x-1)^{24} - \log_5 (x-1)^6 - \log_5 25$

$= \log_5 \frac{(x-1)^{24}}{(x-1)^6} - \log_5 25 = \log_5 (x-1)^{18} - \log_5 25 = \log_5 \frac{(x-1)^{18}}{25}$

7. (8pts) Find the inverse of $f(x) = 4e^{x+3}$.

$y = 4e^{x+3}$

$\frac{y}{4} = e^{x+3}$ | ln

$\ln \frac{y}{4} = x+3$

$x = \ln \frac{y}{4} - 3 = f^{-1}(y)$

8. (14pts) Iodine 131, which decays according to the exponential law and whose half-life is 8 days, was released into the atmosphere as a result of the accident at the nuclear power plant Chernobyl in 1986. This contaminated hay in many parts of Europe.
a) Find the k from the exponential law.
b) If it is all right to feed the hay to cows when 10% of iodine 131 remains, how long do farmers have to wait to use this hay?
c) Without much computation, how long do you have to wait for iodine 131 to decay to $\frac{1}{8}$ of the original amount?

a) $A = A_0 e^{kt}$

$\frac{1}{2}A_0 = A_0 e^{k \cdot 8} \quad | \div A_0$

$\frac{1}{2} = e^{8k} \quad | \ln$

$\ln \frac{1}{2} = 8k$

$k = \frac{\ln \frac{1}{2}}{8} \approx -0.08664$

b) $0.1A_0 = A_0 e^{k \cdot t} \quad | \div A_0$

$0.1 = e^{kt} \quad | \ln$

$\ln 0.1 = k \cdot t$

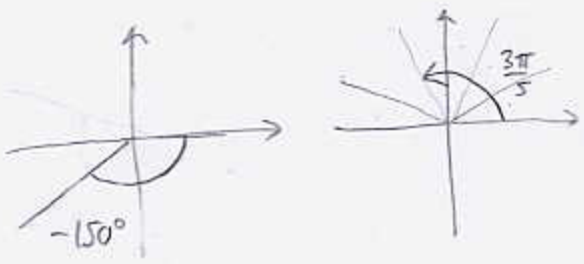
$t = \frac{\ln 0.1}{k}$

≈ 26.5754 days

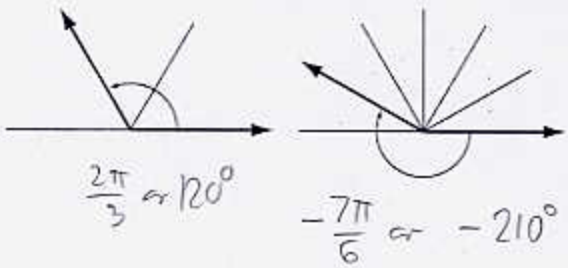
c) $8+8+8 = 24$ days

(Sm G $\frac{1}{8} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$)

9. (4pts) In a coordinate system, roughly draw angles of measure $-\cancel{210}^{-150}$° and $\frac{3\pi}{5}$ radians.



10. (4pts) Indicate both the radian and degree measure under the following angles. (Use equally-spaced auxiliary lines to help you determine what the angles are.)



11. (8pts) The city of Veracruz, Mexico is directly south of Dallas, Texas. If the latitudes of Veracruz and Dallas are approximately 19° and 32.5°, respectively, what is the distance between the two cities? Assume radius of Earth is 3960 miles.

$s = r \cdot \theta = 3960 \cdot 13.5 \cdot \frac{\pi}{180} = 933.053$ miles

$\theta = 32.5 - 19 = 13.5$

12. (4pts) A wheel is rotating with angular speed 48π radians per minute. How many rounds per minute is this?

48π radians per minute $= \frac{48\pi}{2\pi}$ rev. per min $= 24$ rpm

$1 \text{ rev} = 2\pi \text{ rad.}$

Bonus (10pts) Show that $\log_a\left(x+\sqrt{x^2-1}\right)+\log_a\left(x-\sqrt{x^2-1}\right)=0$.

$= \log_a\left(\left(x+\sqrt{x^2-1}\right)\cdot\left(x-\sqrt{x^2-1}\right)\right)$

$= \log_a\left(x^2-\left(\sqrt{x^2-1}\right)^2\right) = \log_a\left(x^2-(x^2-1)\right)$

$= \log_a 1 = 0.$