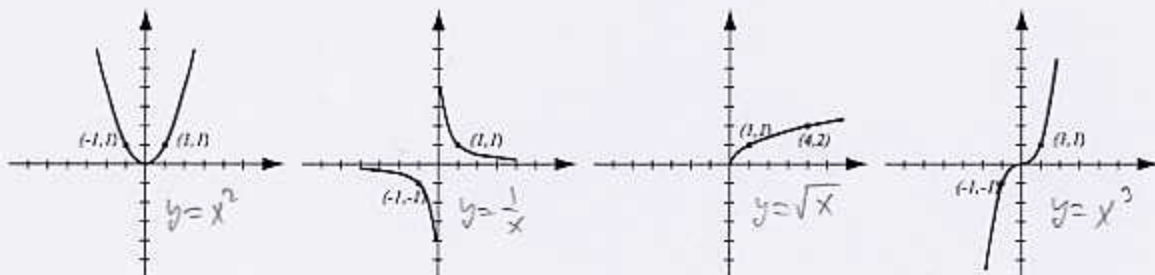1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.

$y = x^2$ $y = \frac{1}{x}$ $y = \sqrt{x}$ $y = x^3$

2. (8pts) Let $f(x) = \dfrac{x^2 - x - 6}{x^2 - 3x - 28}$.

a) Find the domain of this function.

b) Where does the graph of this rational function have vertical asymptotes?
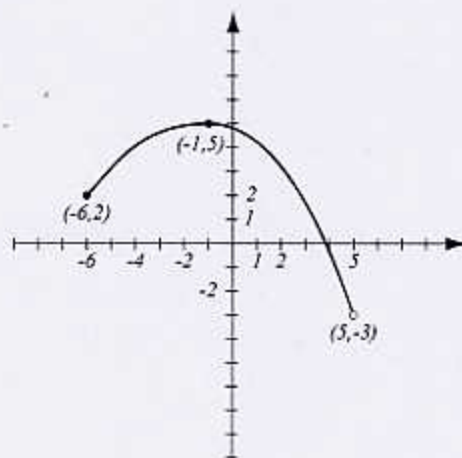
a) denom $= 0$

$x^2 - 3x - 28 = 0$

$(x-7)(x+4) = 0$

$x = 7, -4$

$D = \{x \mid x \neq 7 \text{ and } x \neq -4\}$

b) The rational function has vertical asymptotes where it is not defined

$x = 7, \quad x = -4$

3. (12pts) Use the graph of the function f at right to answer the following questions.

a) What are the x-intercepts?

b) What is the range?

c) Where is the function increasing?

d) Where does f have a local maximum? What is its value?

e) For which x's is $f(x) \leq 0$?

a) $x = 3.8$

b) Range $= (-3, 5]$

c) on interval $(-6, -1)$

d) at $x = -1$ with value $y = 5$

e) on interval $[3.8, 5)$

4. (12pts) "Everything but", a small manufacturer of kitchen sinks, produced 20 sinks in one day at a cost of $3100. On another day, "Everything but" produced 32 sinks at a cost of $3640.

a) Write the cost as a function of number x of sinks produced, assuming it is linear.

b) What is the cost of producing 29 sinks?

a) Need a line through

(20, 3100) and (32, 3640)

$$m = \frac{3640 - 3100}{32 - 20} = \frac{540}{12} = 45$$

$$y - 3100 = 45(x - 20)$$

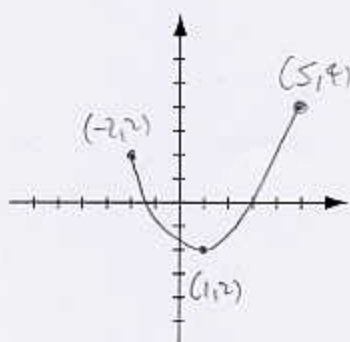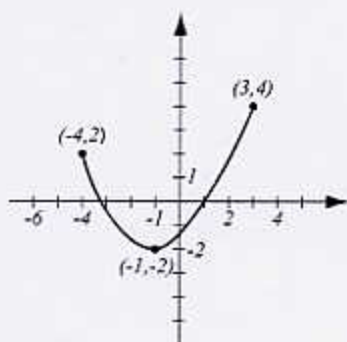$$y = 45x - 900 + 3100$$

$C(x) = 45x + 2200$

b) $C(29) = 3505$

5. (11pts) Let $f(x) = 5 - x$ and $g(x) = x^2 - 1$. Find the following and simplify where possible:

$\frac{f}{g}(2) = \frac{f(2)}{g(2)} = \frac{5-2}{2^2-1} = \frac{3}{3} = 1$
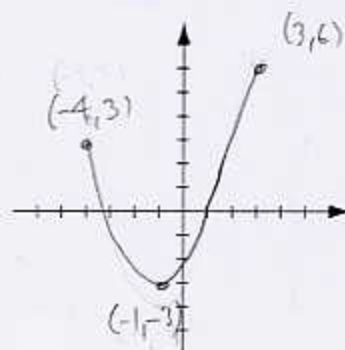
$g(3t + 2) = (3t+2)^2 - 1 = 9t^2 + 12t + 4 - 1 = 9t^2 + 12t + 3$

$f(4 - u^2) = 5 - (4 - u^2) = 1 + u^2$

6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x - 2)$ and $1.5f(x)$ and label all the relevant points.

(-4,2) (3,4) (-1,-2)

(-2,2) (5,4) (1,2)

shift right 2

(-4,3) (3,6) (-1,-3)

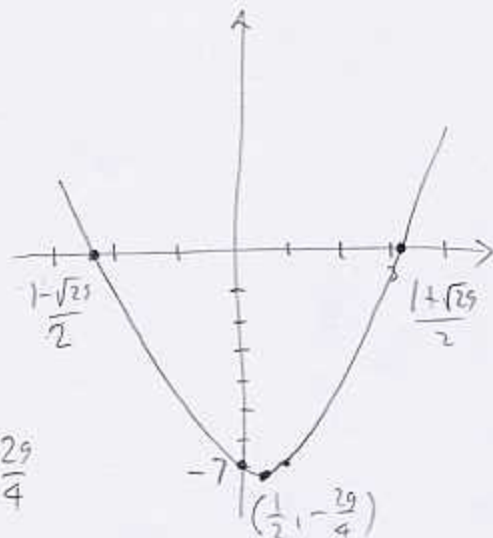stretch vertically, factor = 1.5

7. (12pts) The quadratic function $f(x) = x^2 - x - 7$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) $x^2 - x - 7 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-7)}}{2} = \frac{1 \pm \sqrt{29}}{2}$$

$$= 3.1926, \; -2.1926$$

b) $x = -\frac{-1}{2 \cdot 1} = \frac{1}{2}$

$$y = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 7 = \frac{1}{4} - \frac{1}{2} - 7 = \frac{1 - 2 - 28}{4} = -\frac{29}{4}$$


8. (17pts) Consider the polynomial $f(x) = x^3 - 25x$.
a) Algebraically determine whether this function is even, odd or neither.
b) Algebraically find the x-intercepts of the graph and the y-intercept.
c) Sketch the graph of the function on paper (large, clear, with units — make Dad proud!).
d) Does your graph support the conclusion in a)? Explain why.
e) Use your calculator to determine where the function is decreasing. (Accuracy: 4 decimal points.)

a) $f(-x) = (-x)^3 - 25(-x)$

$\quad = -x^3 + 25x = -f(x)$

it is odd

b) $x^3 - 25x = 0$

$\quad x(x^2 - 25) = 0$

$\quad x = 0 \;$ or $\; x^2 = 25$

$\qquad\qquad x = \pm 5$

$\quad x = 0, \pm 5$

c)
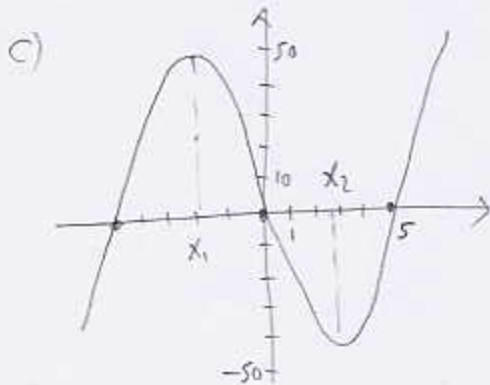
d) yes — it is symmetric about the origin

e) turning pts: $x_1 = -2.8868$
$\qquad\qquad\qquad x_2 = 2.8868$

Decreasing on $(-2.8868, 2.8868)$

9. (10pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 3x - 2, & \text{if } -4 < x < 2 \\ 4 - x, & \text{if } 2 \leq x. \end{cases}$$

| x	3x-2
-4	-14
2	4

x	4-x
2	2
5	-1
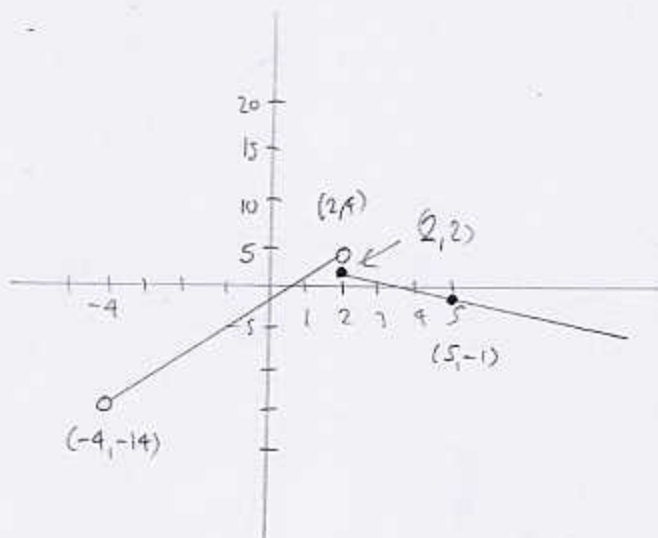
Bonus (10pts) A square piece of cardboard is 10 centimeters wide. To make a box with an open top, a square piece whose side is x is cut from each corner and the remaining flaps are folded upwards.

a) Express the volume V of the box as a function of x. What is the domain of this function?

b) Use your calculator to draw a rough graph of the function $V(x)$.

c) Use your calculator to find the size x of the piece that has to be removed in order to get a box of the greatest volume. What is the maximal volume?

10-2x

10-2x   10-2x
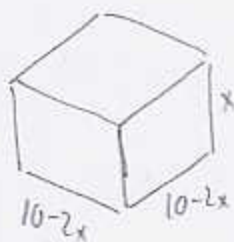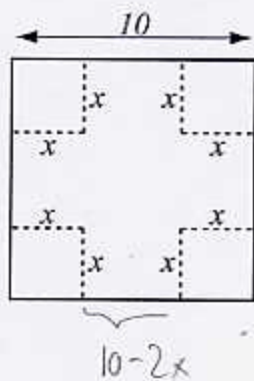
a) $V = (10 - 2x)^2 x$

$0 \leq x \leq 5$ (can't cut more than 5cm on each side)

b)
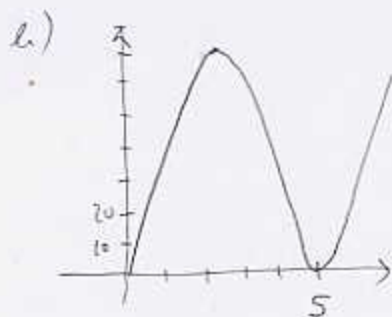
c) max occurs at $x = 1.6667$ cm
with value $y = 74.0741$ cm^3

Max. volume is 74.0741 cm^3