1. (6pts) Solve for x:

$a^2x + b = -a + b^2x$

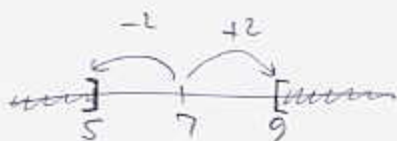$a^2x - b^2x = -a - b$

$x(a^2 - b^2) = -(a+b)$

$x = -\dfrac{a+b}{a^2-b^2} = -\dfrac{a+b}{(a+b)(a-b)} = -\dfrac{1}{b-a}$

2. (8pts) Solve the inequality and write the solution in interval notation: $|x - 7| \geq 2$.

distance from x to $7 \geq 2$

$(-\infty, 5]$ or $[9, \infty)$

3. (12pts) Find the equation of the line that contains $(2, 4)$ and is perpendicular to the line through $A = (-1, 3)$ and $B = (5, -2)$. Sketch both lines on the same coordinate system.

line through $(-1, 3)$ and $(5, -2)$

has slope $m = \dfrac{-2-3}{5-(-1)} = -\dfrac{5}{6}$

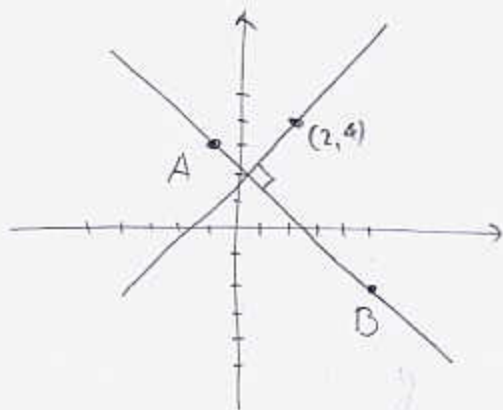Our line has to have

slope $-\dfrac{1}{-\frac{5}{6}} = \dfrac{6}{5}$

Equation: $y - 4 = \dfrac{6}{5}(x - 2)$

$y = \dfrac{6}{5}x - \dfrac{12}{5} + 4$

$y = \dfrac{6}{5}x - \dfrac{8}{5}$

4. (12pts) Put the complex number into form $a + bi$.

a) $\dfrac{(1+i)(3+2i)}{i} = \dfrac{3+3i+2i+2i^2}{i} = \dfrac{1+5i}{i} \cdot \dfrac{-i}{-i} = \dfrac{-i-5i^2}{1} = 5-i$

b) (explain) $i^{31} = i^3 = -i$

$$31 \div 9 = 7, \text{ rem } 3$$

5. (10pts) Let $A = (-1, 4)$, $B = (-3, 8)$.
a) Find the midpoint M of the segment AB.
b) Verify that the distances from M to A and B are equal.

a) $M = \left(\dfrac{-1+(-3)}{2}, \dfrac{4+8}{2} \right) = (-2, 6)$

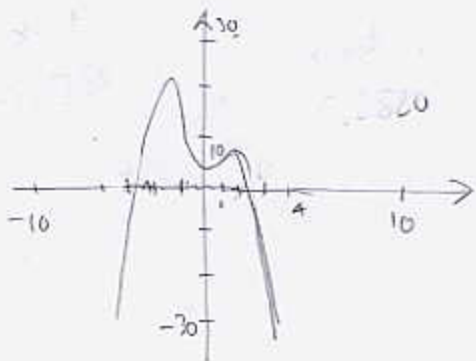b) $d(A,M) = \sqrt{(-1-(-2))^2 + (4-6)^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$

$d(B,M) = \sqrt{(-3-(-2))^2 + (8-6)^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\Rightarrow$ same distance

6. (14pts) The equation $y = -x^3 + 6x^2 - x + 11$ is given.
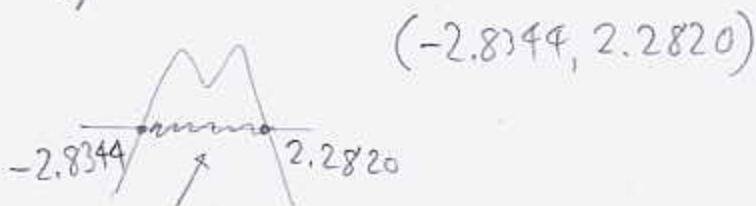a) Use your calculator to accurately sketch the graph of the equation on paper. Indicate your viewing window.
b) What is the y-intercept of the graph?
c) Using your calculator, solve the inequality $-x^3 + 6x^2 - x + 11 > 0$. Write your answer in interval notation with accuracy 4 decimal points.

b) y int is 5

c)

$(-2.8344, 2.2820)$

where curve is above x-axis

Solve the equations, using complex numbers if necessary:

7. (8pts) $x^2 - 7x + 5 = 0$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot 5}}{2} = \frac{7 \pm \sqrt{49 - 20}}{2} = \frac{7 \pm \sqrt{29}}{2}$$

8. (10pts) $x^4 - 3x^2 - 40 = 0$

Let $u = x^2$

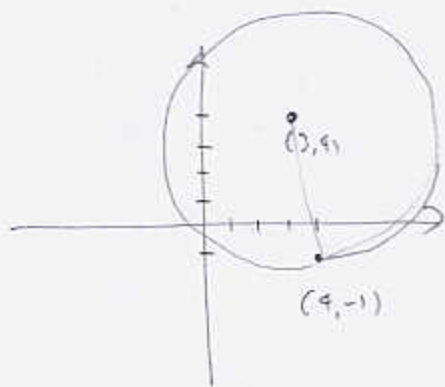$u^2 - 3u - 40 = 0$

$(u-8)(u+5) = 0$

$u = 8, -5$

$x^2 = 8$

$x = \pm\sqrt{8} = \pm 2\sqrt{2}$

$x^2 = -5$

$x = \pm\sqrt{-5} = \pm\sqrt{5}\,i$

9. (8pts) Find the equation of the circle whose center is $(3,4)$ and contains the point $(4,-1)$. Sketch the circle. *(Hint: what is the radius?)*

$$r = d((3,4),(4,-1)) = \sqrt{(4-3)^2 + (-1-4)^2} = \sqrt{1+25} = \sqrt{26}$$

$(x-3)^2 + (y-4)^2 = 26$

is the equation of the circle

10. (12pts) How many liters of water needs to be added to $\overset{4}{\cancel{2}}$ liters of a 15% solution of hydrochloric acid in order to get a 10% solution? Don't forget to write down what your variable means.

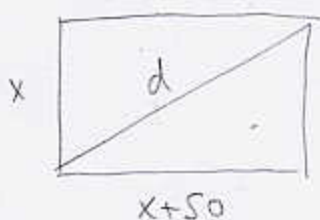$$\overset{x}{\boxed{\text{water}}} + \overset{4\ell}{\boxed{15\%\ Hcl}} = \overset{x+4}{\boxed{10\%\ Hcl}}$$

pure
acid:
$$0 + 0.15 \cdot 4 = 0.1(x+4)$$

$$0.6 = 0.1x + 0.4$$

$$0.2x = 0.1 \quad /\div 0.1$$

$$x = \frac{0.2}{0.1} = 2\ell$$

Bonus (10pts) Farmer Jeremiah has 700ft of fencing. Using all of this fencing, he plans to enclose a rectangular plot of land and divide it into two triangles by stringing fence along one diagonal of the rectangular plot. If the length of the rectangular region has to be 50ft more than the width, what are the dimensions of the enclosure?

let x = width

$x+50$

$d^2 = x^2 + (x+50)^2$

$2x + 2(x+50) + d = 700$

$4x + 100 + d = 700$

$d = 600 - 4x \quad |^\wedge$

dimensions are
103.6608×153.6608 ft

$d^2 = (600 - 4x)^2$

$x^2 + (x+50)^2 = (600-4x)^2$

$x^2 + x^2 + 100x + 2500 = 360000 - 4800x + 16x^2$

$14x^2 - 4900x + 357500 = 0 \quad |\div 2$

$7x^2 - 2450x + 178750 = 0$

$x = \dfrac{2450 \pm \sqrt{2450^2 - 4 \cdot 7 \cdot 178750}}{2 \cdot 7} = \dfrac{2450 \pm \sqrt{997500}}{14}$

$= \dfrac{2450 \pm 998.7492}{14} = 246.3392, \ 103.6608$

too big   solution

already $4 \cdot 246 > 700$