1. (3pts) Sketch angles in standard position with indicated degree angle measure.
$75^{\circ}$
$415^{\circ}$
$-150^{\circ}$
2. (3pts) Sketch angles in standard position with indicated radian angle measure.

$$
\frac{\pi}{3} \quad \frac{5 \pi}{4} \quad-\frac{5 \pi}{6}
$$

3. (8pts) Indicate both the radian and degree measure under the following angles. (Use equally-spaced auxiliary lines to help you determine what the angles are.)

4. (5pts) Convert the angle $63.28^{\circ}$ to
a) $D^{\circ} M^{\prime} S^{\prime \prime}$ form (show work!)
b) radians.
5. (5pts) Nashville, TN ( $36^{\circ}$ north latitude) is almost directly north of Pensacola, FL ( $30^{\circ}$ north latitude). Find the distance between those cities, assuming that the radius of Earth is 3960 miles.
6. (6pts) A car whose tires have radius 22in is traveling at 45 mph . How many revolutions per minute do the wheels make?
7. (5pts) If $\theta$ is an acute angle, find the values of $\cos \theta, \csc \theta$ and $\cot \theta$ given that $\sin \theta=\frac{3}{7}$.
8. (4pts) Use fundamental identities and complementary angles to simplify: $\sin 31^{\circ} \cos 59^{\circ}+\cos 31^{\circ} \sin 59^{\circ}=$
$1+\tan ^{2} 15^{\circ}-\csc ^{2} 75^{\circ}=$
9. (5pts) Find the exact value of each expression. Do not use the calculator.
$\sin ^{2} 60^{\circ}+\tan ^{2} 30^{\circ}=$
$\left(\cos \frac{\pi}{4}-\sin \frac{\pi}{6}\right) \cot \frac{\pi}{3}=$
10. (3pts) Use your calculator to evaluate (round to 4 decimals):
$\cot 223^{\circ}=$

$$
\csc \frac{3 \pi}{7}=
$$

5. (8pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression. $\cos 240^{\circ}=\quad \tan \frac{3 \pi}{2}=\quad \csc \left(-225^{\circ}\right)=\quad \cot \frac{5 \pi}{3}=$
6. (5pts) If $\sin \theta=-\frac{2}{5}$ and $\theta$ is in the third quadrant, find $\cos \theta, \cot \theta, \sec \theta$. Draw a picture.
7. (5pts) Draw two periods of the graph of $y=3 \sin (2 x)$. What is the amplitude? The period? Indicate where the special points are ( $x$-intercepts, peaks, valleys).
8. (5pts) Draw two periods of the graph of $y=-4 \cos (\pi x)$. What is the amplitude? The period? Indicate where the special points are ( $x$-intercepts, peaks, valleys).
9. (5pts) Draw two periods of the graph of $y=\tan \left(\frac{1}{3} x\right)$. What is the period? Indicate where the special points are ( $x$-intercepts, asymptotes).
10. (8pts) For each of the following two graphs, do the following:
a) Find the amplitude.
b) Find the period.
c) Use this information to help you find the equation for each graph.


11. (7pts) a) Use the unit circle to find all the angles where $\csc \theta$ is not defined.
b) What is the period of $\csc \theta$ ?
c) Use your calculator to help you sketch two periods of the graph of $y=\csc \theta$. Indicate where the special points are ( $x$-intercepts, peaks, valleys, asymptotes).
12. (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.
$\arccos \frac{\sqrt{3}}{2}=\quad \arcsin \left(-\frac{\sqrt{2}}{2}\right)=\quad \arcsin (-3)=\quad \arctan \frac{1}{\sqrt{3}}=$
13. (6pts) Use the picture below to estimate (in degrees) the values of inverse trigonometric functions. Compare your answer with results you get with a calculator.

estimate calculator
$\arcsin 0.8=$
$\arccos (-0.4)=$
$\arctan (-1.5)=$
14. (5pts) Simplify the following expressions without using the calculator. For some of them, you will need a picture.
$\sin (\arcsin (0.83))=$
$\arctan \left(\tan \frac{2 \pi}{7}\right)=$
$\arccos \left(\cos \frac{6 \pi}{5}\right)=$
15. (7pts) Evaluate the following expressions exactly. Draw the appropriate picture.
$\sin \left(\arctan \frac{5}{4}\right)=$
$\tan \left(\arccos \left(-\frac{2}{3}\right)\right)=$
16. (4pts) A 13 ft ladder is leaning against the wall. If its bottom is 4 ft away from the wall, what is the angle (in degrees) between the ladder and the ground?
17. (6pts) Suppose that $\pi<\alpha<\frac{3 \pi}{2}$ and $-\frac{\pi}{2}<\beta<0$ are angles so that $\cos \alpha=-\frac{2}{3}$ and $\cos \beta=\frac{1}{3}$. Find the exact value of $\sin (\alpha+\beta)$.
18. (4pts) Find the exact value of $\tan 22.5^{\circ}$ (do not use the calculator).
19. (3pts) Find the exact value of the expression (do not use the calculator): $\cos 22^{\circ} \cos 52^{\circ}+\sin 22^{\circ} \sin 52^{\circ}=$

Show the following identities.
4. (4pts) $\frac{\sin (\alpha+\beta)}{\cos \alpha \cos \beta}=\tan \alpha+\tan \beta$
5. (5pts) $\sin ^{2} \theta \cos ^{2} \theta=\frac{1}{8}(1-\cos (4 \theta))$
6. (8pts) Find the exact value (do not use the calculator) of $\cos \frac{5 \pi}{12}$ in two ways. (Your answers may differ how they look, but they should be the same number, which you may check using the calculator.)
a) Use the addition formula.
b) Use a half-angle formula.

## Spring '06/MAT 145/Worksheet 6 Name:

1. (4pts) Solve the equation (give a general formula for all the solutions).
$2 \sin \theta+1=0$
2. (8pts) Solve the equation and give a general formula for all the solutions. Then list all the solutions that fall in the interval $-\frac{3 \pi}{2} \leq \theta \leq \frac{3 \pi}{2}$.
$\cos (2 \theta)=\frac{\sqrt{2}}{2}$
3. (5pts) Use your calculator to solve the equation on the interval $0 \leq \theta \leq 2 \pi$. Round answers to two decimal places (answers in radians).
$\cos \theta=0.4$
4. (7pts) Solve the equation (give a general formula for all the solutions).
$2 \sin ^{2} \theta+11 \sin \theta-6=0$
5. ( 6 pts ) Solve the equation on the interval $0 \leq \theta \leq 2 \pi$.
$\sqrt{3} \sin \theta+\cos \theta=0$
6. (6pts) A spotlight sits on the ground and shines at a wall 40 ft away. If a 6 ft man stands between the spotlight and the wall, his shadow on the wall is 13 ft tall.
a) How far from the spotlight is the man standing?
b) What is the angle of elevation of the line connecting the spotlight with the top of the man's shadow?
7. (8pts) To find straight-line distance across a pond, surveyors took some measurements that are indicated in the picture.
a) Find the angle at $C$.
b) Find the distance from $A$ to $C$.
c) Find the distance from $B$ to $C$.

8. (7pts) Solve the triangle: $a=6, \alpha=47^{\circ}, \gamma=56^{\circ}$.
9. (9pts) A solar panel rests on a 12 ft beam. To maximize efficiency of the solar panel, the beam must have an angle of $38^{\circ}$ with the ground. Another 8 ft beam will be used to support the structure by attaching one end of it to the free end of the 12 ft beam, and the other end will be anchored in the ground.
a) Investigate all the ways this can be done without cutting the 8 ft beam.
b) How far will the possible anchor points of the 8 ft beam be away from the anchor point of the 12 ft one?

10. (6pts) Solve the triangle: $b=5, c=8, \alpha=37^{\circ}$.
11. ( 8 pts ) Consider the pictured trapezoid (top and bottom sides are parallel).
a) Find the length and angle to the horizontal of the left slanted side.
b) Find the area of the trapezoid.

12. (8pts) Solve the triangle: $a=4, b=7, c=6$. Then find its area in two ways: by using one of the angles and by using Heron's formula. Did you get the same answer?
13. (8pts) In attempting to fly from Chicago to Louisville, a distance of 330 miles, a pilot took a course that was $10^{\circ}$ in error.
a) If the aircraft averaged 220 mph and if the error is discovered after 15 minutes, through what angle should the pilot turn to head toward Louisville?
b) What new average speed should the pilot maintain so that the total time of the trip is 90 minutes?

