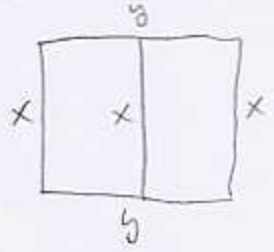
**Spring '06/MAT 140/Worksheet 8** **Name:** Solution *Show all your work.*

**1.** (8pts) Anna has 4000ft of fencing with which she plans to enclose a rectangular field and divide it into two parts by running some of the fence across the field, parallel to the width side (picture).
a) Express the area $A$ of the rectangle as a function of the width $x$ of the rectangle.
b) Draw an accurate graph of the function $A(x)$.
c) For what $x$ is the area the largest? What is the maximum area?




$3x+2y=4000$

$y=\frac{4000-3x}{2}=2000-\frac{3}{2}x$

$A=xy=x\left(2000-\frac{3}{2}x\right)$

$=-\frac{3}{2}x^2+2000x$

b) intercepts: $x(2000-\frac{3}{2}x)=0$

$x=0$ or $2000-\frac{3}{2}x=0$

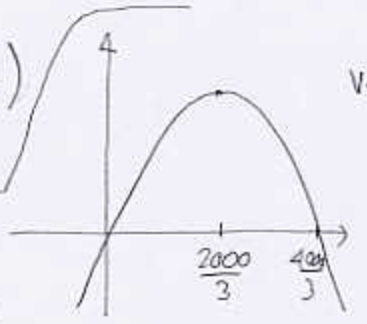
$\frac{3}{2}x=2000$

$x=\frac{4000}{3}$

Vertex: $x=-\frac{b}{2a}=-\frac{2000}{-2\cdot\frac{1}{2}}=\frac{2000}{3}$

$y=\frac{2000}{3}\left(2000-\frac{3}{2}\cdot\frac{2000}{3}\right)$

$=\frac{2000}{3}\cdot 1000=\frac{2,000,000}{3}=666,666.67$




c) Max. area of 666,666.67
occurs for $x=\frac{2000}{3}=666.67$ ft

**2.** (11pts) Consider the polynomial $P(x)=(x+2)(x+4)(x-5)^2$. Answer the following (decimal answers should have accuracy to two decimal places).
a) Find the $x$-intercepts of the graph and the $y$-intercept.
b) $P$ behaves like what function for large $|x|$?
c) Find the turning points of $P$.
d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.
e) Use the graph to determine the range of the function.

a) $(x+2)(x+4)(x-5)^2=0$

$x=-2,-4,5$ ← x-int

$P(0)=2\cdot 4\cdot(-5)^2=200$

b) $P(x)=(x^2+6x+8)(x^2-10x+25)$

$=x^4+$ smaller powers

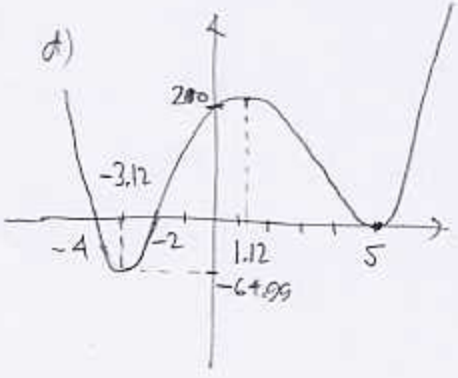
behaves like $x^4$

c) Turning points: $x=-3.12$, $y=-64.99$

$x=1.12$ $y=240.49$

$x=5$, $y=0$

d)




e) Range is $[-64.99,\infty)$

$[-10,10]\times[-300,300]$

**3.** (11pts) Consider the rational function $Q(x) = \dfrac{2x^2 - 5}{x^2 - x - 12}$. Answer the following (decimal answers should have accuracy to two decimal places).

a) Find the domain of the function and where the vertical asymptotes are.

b) Find the $x$-intercepts of the graph and the $y$-intercept.

c) Find the horizontal asymptote, if any.

d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.

e) Find the intervals where the function is decreasing.

a) $x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ or $x = -3$

Vertical asymp. $x = 4, x = -3$

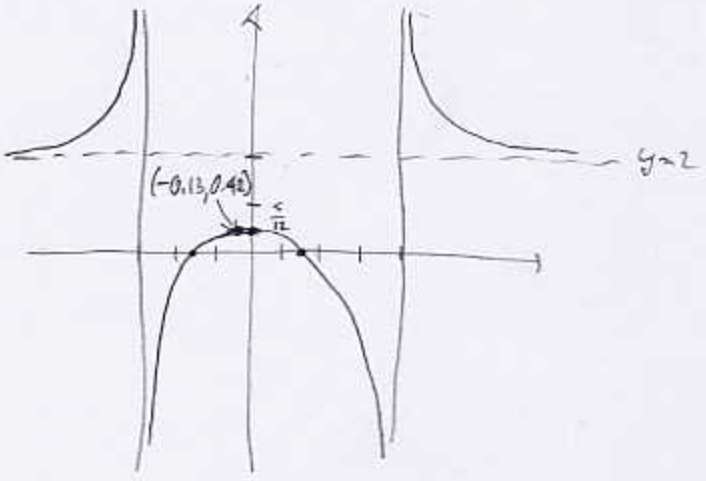
b) $2x^2 - 5 = 0$

$x^2 = \frac{5}{2}$

$x = \pm\sqrt{\frac{5}{2}} \approx \pm 1.58$

y-int: $Q(0) = \frac{-5}{-12} = \frac{5}{12}$

c) $y = \frac{2}{1} = 2$ is horizontal asymp.




e) Need turning point: $x = -0.13$

$y = 0.42$

decreasing on $(-0.13, 4)$ and $(4, \infty)$