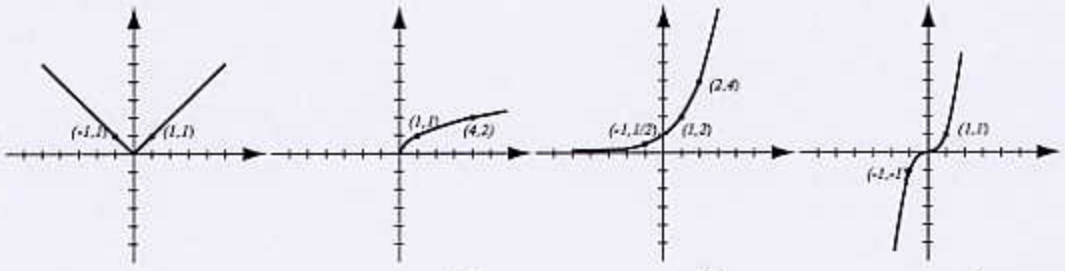
Spring '06/MAT 140/Final exam **Name:** Solution *Show all your work.*

1. (4pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = |x|$ $y = \sqrt{x}$ $y = 2^x$ $y = x^3$

2. (4pts) Find the equation of the line that passes through $(3, -2)$ and is perpendicular to the line $3x + 2y = 7$. Draw both lines in the same coordinate system.

$3x + 2y = 7$

$2y = -3x + 7 \quad | \div 2$

$y = -\frac{3}{2}x + \frac{7}{2}$

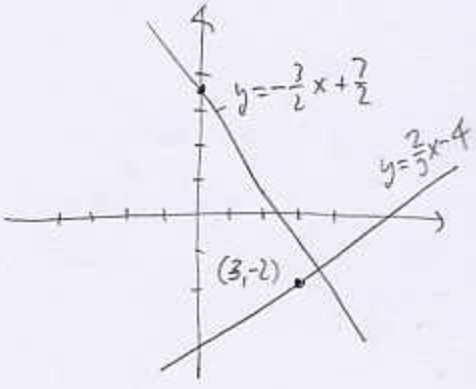
slope of perp. line is $-\frac{1}{-\frac{3}{2}} = \frac{2}{3}$

Equation of perp. line:

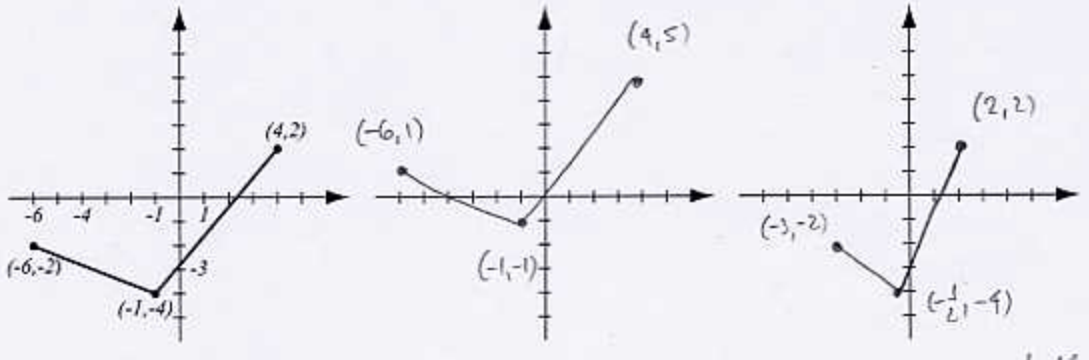
$y - (-2) = \frac{2}{3}(x - 3)$

$y = \frac{2}{3}x - \frac{2}{3}\cdot 3 - 2$

$y = \frac{2}{3}x - 4$



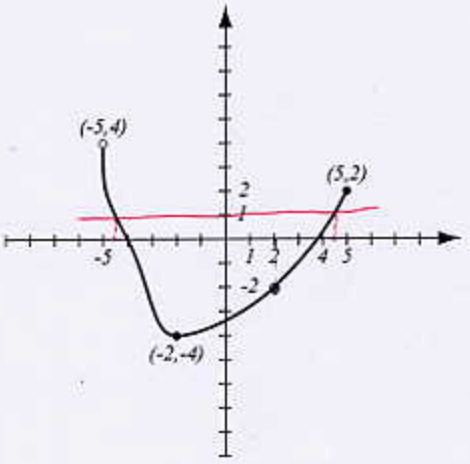
3. (4pts) The graph of the function f is given below. On separate graphs, sketch the graphs of the functions $f(x) + 3$ and $f(2x)$. Label all the relevant points.



shift up 3

stretch horizontally by factor $\frac{1}{2}$

4. (7pts) Use the graph of the function f at right to answer the following questions.



a) Find $f(2)$. $f(2) = -2$

b) What is the range of f? $[-4, 4)$

c) List the x-intercepts of the graph. $x = -4$
$x = 3.75$

d) Where does f have a local minimum? What is its value? Local min at $x = -2$ with value $y = -4$

e) What are the solutions of the equation $f(x) = 1$?
$x = -4.5$
$x = 4.5$

5. (7pts) The quadratic function $f(x) = x^2 - 2x - 6$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) $x^2 - 2x - 6 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot (-6)}}{2}$$

$$= \frac{2 \pm \sqrt{28}}{2}$$

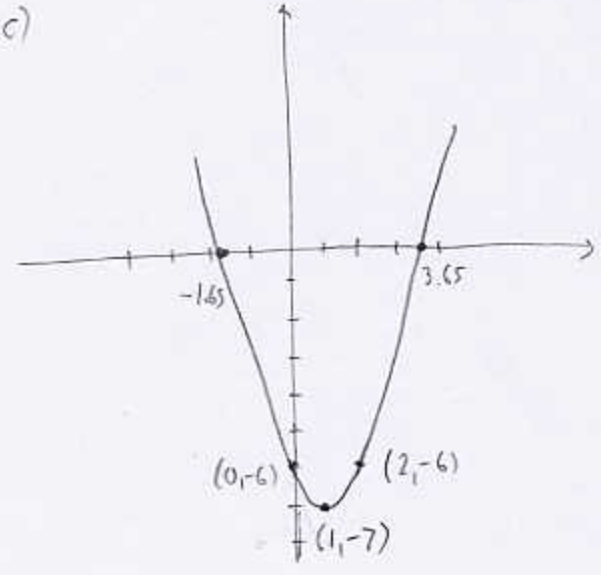
$$= \frac{2 \pm 2\sqrt{7}}{2}$$

$$= 1 \pm \sqrt{7} = 3.65, -1.65$$

b) $x = -\frac{b}{2a} = -\frac{-2}{2 \cdot 1} = 1$

$y = 1^2 - 2 \cdot 1 - 6 = -7$

c)



6. (7pts) Consider the polynomial $P(x) = 4(x-3)^2(x+1)$. Answer the following (decimal answers should have accuracy to two decimal places).
a) Find the x-intercepts of the graph and the y-intercept.
b) P behaves like what function for large $|x|$?
c) Find the turning points of P.
d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.

a) $x = 3, -1$ x-int.

$y = 4(-3)^2 \cdot 1 = 36$

c) (3,0) and (0.33, 37.93)

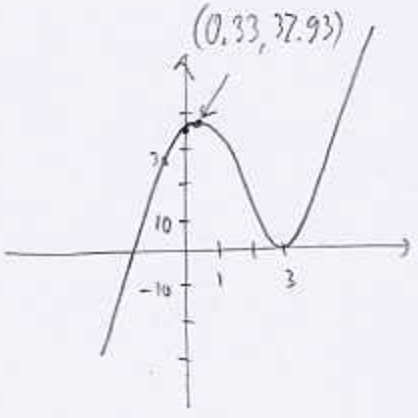
b) $4(x-3)^2(x+1)$

$= 4(x^2 + \_)(x+1)$

$= 4x^3 +$ lower powers

Behaves like $4x^3$

d)



7. (4pts) Simplify and write the answer so all exponents are positive:

$$\frac{(2x)^4(x^{-3}y^5)^3}{(xy)^{-4}(10y)^2} = \frac{16x^4 x^{-9} y^{15}}{x^{-4} y^{-4} 100 y^2} = \frac{4x^{-5} y^{15}}{25 x^{-4} y^{-2}} = \frac{4y^{15-(-2)}}{25x^{-4-(-5)}} = \frac{4y^{17}}{25x}$$

8. (4pts) Simplify.

$$\frac{x+1}{x^2+4x-5} + \frac{2x-1}{x^2+10x+25} = \frac{x+1}{(x+5)(x-1)} + \frac{2x-1}{(x+5)^2} = \frac{(x+1)(x+5)+(2x-1)(x-1)}{(x+5)^2(x-1)}$$

$$= \frac{x^2+6x+5+2x^2-3x+1}{(x+5)^2(x-1)} = \frac{3x^2+3x+6}{(x+5)^2(x-1)}$$

9. (4pts) Let $f(x) = \dfrac{2x}{5x-1}$.

a) Find $f^{-1}(x)$.

b) Find the range of f.

$y = \dfrac{2x}{5x-1}$ $\quad x = \dfrac{y}{5y-2}$

$(5x-1)y = 2x$

$5xy - y = 2x$

$5xy - 2x = y$

$x(5y-2) = y$

$f^{-1}(y) = \dfrac{y}{5y-2}$

Range of f = Domain of f^{-1}

$= \{y \mid y \neq \frac{2}{5}\}$

$5y - 2 = 0$

$y = \frac{2}{5}$

10. (4pts) Solve the equation. $e^{x+3} = 4^{2x-1}$

$e^{x+3} = 4^{2x-1} \mid \ln$

$\ln e^{x+3} = \ln 4^{2x-1}$

$x+3 = (2x-1)\ln 4$

$x+3 = 2x\ln 4 - \ln 4$

$3 + \ln 4 = 2x\ln 4 - x$

$3 + \ln 4 = x(2\ln 4 - 1)$

$x = \dfrac{3+\ln 4}{2\ln 4 - 1} = 2.47$

11. (3pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_5 \dfrac{(x+1)^3}{25} =$ $\log_5 (x+1)^3 - \log_5 25 = 3\log_5(x+1) - 2$

$\log_5 25 = 2$

$5^2 = 25$

12. (6pts) How many milliliters of a 10% solution of muriatic acid needs to be added to 150ml of a 40% solution in order to get a 30% solution? Don't forget to write down what your variable means.

x = amt of 10% solution

$0.1x + 0.4 \cdot 150 = 0.3(x+150)$

$0.1x + 60 = 0.3x + 45$

$15 = 0.2x$

$x = \frac{15}{0.2} = 75 \text{ ml}$

Need 75 ml of 10% solution.

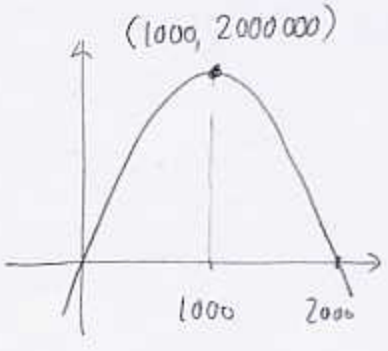
13. (7pts) Sharon has 4000m of fencing and wishes to enclose a rectangular field that borders a river. If she does not fence the side along the river, what is the largest area that can be enclosed?

$A = xy = x(4000-2x) = -2x^2 + 4000x$

quadratic function

$2x + y = 4000$

$y = 4000 - 2x$



$\text{vertex} = -\frac{b}{2a} = -\frac{4000}{2(-2)} = 1000$

corresponding area is

$1000 \cdot (4000 - 2 \cdot 1000)$

$= 2000000 \text{ m}^2$

14. (5pts) Solve the system of equations:

$$\begin{cases} 2x + 6y + 2z = -2 \\ -3x + y + 2z = -2 \\ 5x + 15y - 3z = 3 \end{cases}$$

$$\begin{bmatrix} 2 & 6 & 2 & -2 \\ -3 & 1 & 2 & -2 \\ 5 & 15 & -3 & 3 \end{bmatrix} \xrightarrow{\div 2} \begin{bmatrix} 1 & 3 & 1 & -1 \\ -3 & 1 & 2 & -2 \\ 5 & 15 & -3 & 3 \end{bmatrix} \xrightarrow{\cdot 3,\ \cdot -5} \begin{bmatrix} 1 & 3 & 1 & -1 \\ 0 & 10 & 5 & -5 \\ 0 & 0 & -8 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 1 & -1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$x + 3y + z = -1$ $\quad x = -1 - 3\cdot 0 - (-1) = -1 + 1 = 0$

$y + \frac{1}{2}z = -\frac{1}{2}$ $\quad y = -\frac{1}{2} + \frac{1}{2}\cdot 1 = 0$

$z = -1$

$x = 0,\ y = 0,\ z = -1$

Bonus (7pts) The city of Semesdunn, OK, had 32,000 people six years ago and has 51,000 today. Assume the population grows according to the exponential law, $P(t) = P_0 e^{kt}$, $k > 0$.
a) Find k for this situtation.
b) Assuming growth continues according to the exponential law, what will the population be in four years?

a) $P(t) = 32e^{kt}$ thousand people

$51 = P(6) = 32e^{k\cdot 6}$

$\frac{51}{32} = e^{k\cdot 6}$ $\quad | \ln$

$\ln \frac{51}{32} = k\cdot 6$

$k = \frac{\ln \frac{51}{32}}{6} = 0.0776816212$

b) Population 4 years from now is 10 years from start

$P(10) = 32e^{0.0776816212\cdot 10}$

$= 69.585$

In four years it will be 69,585.