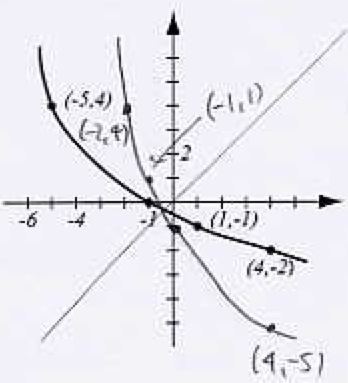
Spring '06/MAT 140/Exam 4b **Name:** Solution *Show all your work.*

1. (4pts) The graph of a function f is given.
a) Explain why the function has an inverse.
b) Find the graph of f^{-1}, labeling the relevant points.



a) because it passes the horizontal line test

2. (4pts) Let $f(x) = \dfrac{3x-7}{4x}$.
a) Find $f^{-1}(x)$.
b) Find the range of f^{-1}.

Range of f^{-1} = Domain of f = $\{x \mid x \neq 0\}$

a) $y = \dfrac{3x-7}{4x}$

$4xy = 3x - 7$

$4xy - 3x = -7$

$x(4y-3) = -7$

$x = -\dfrac{7}{4y-3}$

$f^{-1}(y) = -\dfrac{7}{4y-3}$

3. (4pts) Evaluate without using the calculator:

$\log_2 32 = 5$ $\quad$ $\log_3 \frac{1}{27} = -3$ $\quad$ $\log_9 3 = \dfrac{1}{2}$ $\quad$ $\log_a \sqrt[3]{a^4} = \dfrac{4}{3}$

$2^{?} = 32$ $\quad$ $3^{?} = \dfrac{1}{27}$ $\quad$ $9^{?} = 3$ $\quad$ $a^{?} = a^{\frac{4}{3}}$

4. (3pts) What is the domain of the function $f(x) = \log_4(x-7)$?

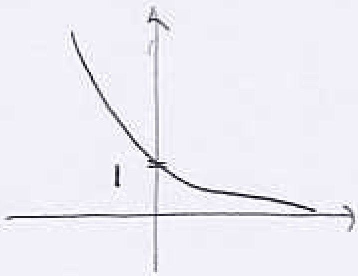
Must have $x - 7 > 0$

$x > 7$

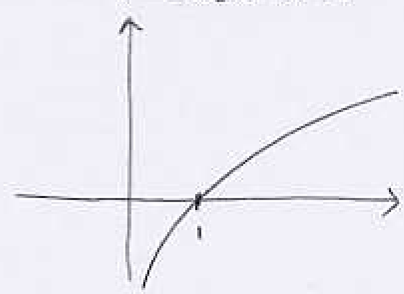
Domain = $(7, \infty)$

5. (3pts) Draw the general shape of the graph for these functions. Indicate the x- and y-intercepts.

$y = a^x$, $a < 1$



$y = \log_a x$, $a > 1$.



Solve the equations:

6. (2pts) $\log_x 3 = 2$

$x^2 = 3$ Only $x=\sqrt{3}$ applies

$x = \pm\sqrt{3}$ since base must be positive.

7. (4pts) $2^{x^2} = 4^{12-x}$

$2^{x^2} = (2^2)^{12-x}$

$2^{x^2} = 2^{24-2x}$

$x^2 = 24-2x$

$x^2+2x-24=0$

$(x+6)(x-4)=0$

$x = -6, 4$

8. (5pts) Solve and then use the calculator to find the decimal value for x.

$3^x = 4^{5x+1}$ $\mid \cdot \ln$

$\ln 3^x = \ln 4^{5x+1}$

$x \ln 3 = (5x+1) \ln 4$

$x \ln 3 = 5x \ln 4 + \ln 4$

$x \ln 3 - 5x \ln 4 = \ln 4$

$x(\ln 3 - 5\ln 4) = \ln 4$

$$x = \frac{\ln 4}{\ln 3 - 5\ln 4} = -0.24$$

9. (2pts) Use your calculator to find $\log_7 0.25$ with accuracy 4 decimal places. Show how you obtained your number.

$$\log_7 0.25 = \frac{\log 0.25}{\log 7} = \frac{-0.6020\ldots}{0.8450\ldots} = -0.7124$$

10. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_7(49x^2\sqrt[4]{y^7-8}) = \log_7 49 + \log_7 x^2 + \log_7 (y^7-8)^{\frac{1}{4}}$

$= 2 + 2\log_7 x + \frac{1}{4}\log_7 (y^7-8)$

$\ln \frac{(x+9)^5}{e^3} = \ln (x+9)^5 - \underbrace{\ln e^3}_{=3} = 5\ln(x+9) - 3$

11. (6pts) Write each the following as a single logarithm. Simplify if possible.

$3\log x^4 + 4\log\sqrt{x} = \log (x^4)^3 + \log(\sqrt{x})^4 = \log x^{12} + \log x^2 = \log x^{12}\cdot x^2 = \log x^{14}$

$(x^{\frac{1}{2}})^4 = x^2$

$3\ln(x+2) - \ln(x^2-x-6) = \ln (x+2)^3 - \ln(x^2-x-6)$

$= \ln \frac{(x+2)^3}{x^2-x-6} = \ln \frac{(x+2)^3}{(x+2)(x-3)} = \ln \frac{(x+2)^2}{x-3}$

12. (7pts) One of the radioactive elements released into the air after the accident at Chernobyl (20 years ago this week) was iodine 131, whose half-life is 8 days. The function describing the decay of iodine 131 is $A(t) = A_0 e^{kt}$, $k < 0$.
a) Find the k for iodine 131.
b) Livestock feed contaminated by iodine 131 is deemed safe for animal consumption once 10% of the original amount of iodine 131 remains. How long after contamination is it OK to use the feed?

a) $\frac{A_0}{2} = A_0 e^{k \cdot 8} \quad | \div A_0$

$\frac{1}{2} = e^{k \cdot 8} \quad | \ln$

$\ln \frac{1}{2} = k \cdot 8$

$k = \frac{\ln(\frac{1}{2})}{8}$

$k = -0.086643...$

≈ -0.0866

b) $0.1 A_0 = A_0 e^{-0.0866 \cdot t} \quad | \div A_0$

$0.1 = e^{kt} \quad | \ln$

$\ln 0.1 = kt$

$t = \frac{\ln 0.1}{k} = 26.58$ days

Bonus. (5pts) The probability that a car will pull up to a bank's drive-through within t minutes of 1PM is modeled by the formula $P(t) = 1 - e^{-0.2t}$. Solve the following with accuracy 2 decimal points.
a) What is the probability that a car will come within 5 minutes of 1PM?
b) How many minutes are needed for probability to reach 99%?

a) $P(5) = 1 - e^{-0.2 \cdot 5}$

$= 1 - e^{-1}$

$= 0.63$

63% probability

b) $0.99 = 1 - e^{-0.2t}$

$-0.01 = -e^{-0.2t}$

$0.01 = e^{-0.2t} \quad | \ln$

$\ln 0.01 = -0.2t$

$t = \frac{\ln 0.01}{-0.2} \approx 23.03$ minutes