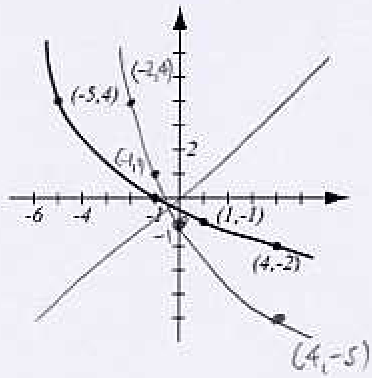
Spring '06/MAT 140/Exam 4a Name: Solution *Show all your work.*

1. (4pts) The graph of a function f is given.
a) Explain why the function has an inverse.
b) Find the graph of f^{-1}, labeling the relevant points.



a) Because f passes the horizontal line test

2. (4pts) Let $f(x) = \dfrac{3x}{7x-4}$.
a) Find $f^{-1}(x)$.
b) Find the range of f^{-1}.

a) $y = \dfrac{3x}{7x-4}$

$y(7x-4) = 3x$

$7xy - 4y = 3x$

$7xy - 3x = 4y$

$x(7y-3) = 4y$

$x = \dfrac{4y}{7y-3}$

$f^{-1}(y) = \dfrac{4y}{7y-3}$

b) Range of f^{-1} = Domain of f

$= \{x \mid x \neq \frac{4}{7}\}$

$7x - 4 = 0$

$x = \frac{4}{7}$

3. (4pts) Evaluate without using the calculator:

$\log_3 27 = 3$ $\qquad \log_2 \frac{1}{32} = -5$ $\qquad \log_{25} 5 = \frac{1}{2}$ $\qquad \log_a \sqrt[7]{a^2} = \frac{2}{7}$

$3^{?} = 27$ $\qquad 2^{?} = \frac{1}{32}$ $\qquad 25^{?} = 5$ $\qquad a^{?} = a^{\frac{2}{7}}$

4. (3pts) What is the domain of the function $f(x) = \log_5(x+4)$?

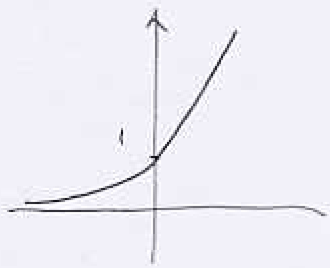
Must have $x + 4 > 0$

$x > -4$

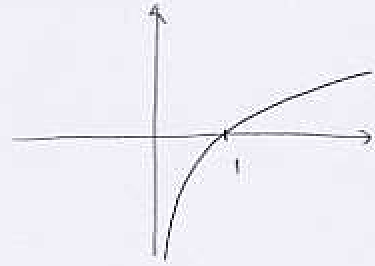
$D = \{x \mid x > -4\} = (-4, \infty)$

5. (3pts) Draw the general shape of the graph for these functions. Indicate the x- and y-intercepts.

$y = a^x$, $a > 1$



$y = \log_a x$, $a > 1$.



Solve the equations:

6. (2pts) $\log_3 x = 2$

$3^2 = x$

$x = 9$

7. (4pts) $5^{x^2+3x} = 125^{2x+6}$

$5^{x^2+3x} = (5^3)^{2x+6}$

$5^{x^2+3x} = 5^{6x+18}$

$x^2+3x = 6x+18$

$x^2-3x-18=0$

$(x-6)(x+3)=0$

$x = 6, -3$

8. (5pts) Solve and then use the calculator to find the decimal value for x.

$7^{3x+4} = 5^{2x}$ $\mid \ln$

$\ln 7^{3x+4} = \ln 5^{2x}$

$(3x+4)\ln 7 = 2x \ln 5$

$3x\ln 7 + 4\ln 7 = 2x\ln 5$

$3x\ln 7 - 2x\ln 5 = -4\ln 7$

$x(3\ln 7 - 2\ln 5) = -4\ln 7$

$x = \dfrac{-4\ln 7}{3\ln 7 - 2\ln 5} = \dfrac{4\ln 7}{2\ln 5 - 3\ln 7} = -2.97$

9. (2pts) Use your calculator to find $\log_9 0.4$ with accuracy 4 decimal places. Show how you obtained your number.

$$\log_9 0.4 = \frac{\log 0.4}{\log 9} = -0.4170$$

10. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4 \frac{(x-2)^6}{16} =$ $\log_4 (x-2)^6 - \log_4 16 = 6 \log_4 (x-2) - 2$

$\ln(e^2 x^{11} \sqrt{2y+3}) =$ $\ln e^2 + \ln x^{11} + \ln (2y+3)^{\frac{1}{2}}$

$= 2 + 11 \ln x + \frac{1}{2} \ln(2y+3)$

11. (6pts) Write each the following as a single logarithm. Simplify if possible.

$2\log x^3 + 6 \log \sqrt{x} =$ $\log (x^3)^2 + \log(\sqrt{x})^6 \leftarrow (x^{\frac{1}{2}})^6 = x^3$

$= \log x^6 + \log x^3 = \log(x^6 \cdot x^3) = \log x^9$

$2\ln(x-3) - \ln(x^2 - x - 6) =$ $\ln (x-3)^2 - \ln (x^2-x-6)$

$= \ln \frac{(x-3)^2}{x^2-x-6} = \ln \frac{(x-3)^2}{(x-3)(x+2)} = \ln \frac{x-3}{x+2}$

12. (7pts) One of the radioactive elements released into the air after the accident at Chernobyl (20 years ago this week) was iodine 131, whose half-life is 8 days. The function describing the decay of iodine 131 is $A(t) = A_0 e^{kt}$, $k < 0$.
a) Find the k for iodine 131.
b) Livestock feed contaminated by iodine 131 is deemed safe for animal consumption once 10% of the original amount of iodine 131 remains. How long after contamination is it OK to use the feed?

a) $\frac{A_0}{2} = A_0 e^{k \cdot 8} \quad | \div A_0$

$\frac{1}{2} = e^{k \cdot 8} \quad | \ln$

$\ln \frac{1}{2} = k \cdot 8$

$k = \frac{\ln(\frac{1}{2})}{8}$

$k = -0.086643...$

≈ -0.0866

b) $0.1 A_0 = A_0 e^{-0.0866 \cdot t} \quad | \div A_0$

$0.1 = e^{kt} \quad | \ln$

$\ln 0.1 = kt$

$t = \frac{\ln 0.1}{k} = 26.58$ days

Bonus. (5pts) The probability that a car will pull up to a bank's drive-through within t minutes of 1PM is modeled by the formula $P(t) = 1 - e^{-0.2t}$. Solve the following with accuracy 2 decimal points.
a) What is the probability that a car will come within 5 minutes of 1PM?
b) How many minutes are needed for probability to reach 99%?

a) $P(5) = 1 - e^{-0.2 \cdot 5}$

$= 1 - e^{-1}$

$= 0.63$

63% probability

b) $0.99 = 1 - e^{-0.2t}$

$-0.01 = -e^{-0.2t}$

$0.01 = e^{-0.2t} \quad | \ln$

$\ln 0.01 = -0.2t$

$t = \frac{\ln 0.01}{-0.2} \approx 23.03$ minutes