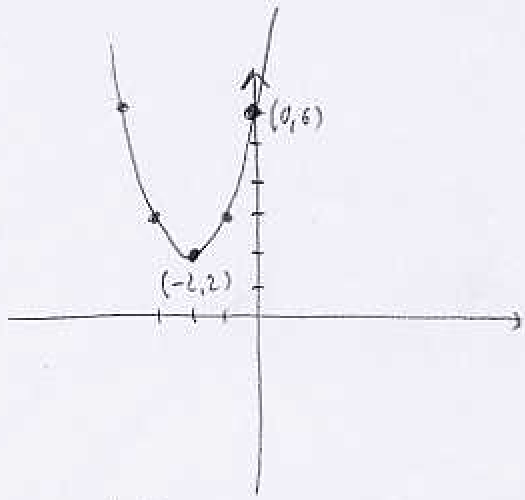
Spring '06/MAT 140/Exam 3b **Name:** Solution *Show all your work.*

1. (8pts) The quadratic function $f(x) = x^2 + 4x + 6$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.
d) What is the range of the function?



a) $x^2+4x+6=0$

doesn't factor

$$x=\frac{-4\pm\sqrt{4^2-4\cdot 6\cdot 1}}{2}=\frac{-4\pm\sqrt{-8}}{2}$$

no solutions — no x-int

d) Range is $[2,\infty)$

b) $x=-\frac{b}{2a}=-\frac{4}{2\cdot 1}=-2$

$y=(-2)^2+4(-2)+6=2$

2. (2pts) The table gives values of f and g for some x's. Find $(g\circ f)(3)$ and $(f\circ f)(1)$.

x	1	2	3
$f(x)$	2	3	2
$g(x)$	2	1	3

$(g\circ f)(3)=g(f(3))=g(2)=1$

$(f\circ f)(1)=f(f(1))=f(2)=3$

3. (5pts) Let $f(x) = 3x + 5$ and $g(x) = \sqrt{x-7}$. Find the following composites (simplify if possible):

$(g\circ f)(x) = g(f(x))$
$=g(3x+5)$
$=\sqrt{(3x+5)-7}$
$=\sqrt{3x-2}$

$(f\circ f)(x) = f(f(x))$
$=f(3x+5)$
$=3(3x+5)+5$
$=9x+20$

4. (3pts) Let $h(x) = \dfrac{2}{x^2+1}$. Break up this function into a composite of two functions f and g. That is, find f and g so that $h(x) = (f \circ g)(x)$.

To compute $\dfrac{2}{x^2+1}$

1) find x^2+1

2) divide 2 by result

$g(x) = x^2+1$

$f(x) = \dfrac{2}{x}$

5. (11pts) Consider the polynomial $P(x) = -(x-3)(x+4)(x-5)$. Answer the following (decimal answers should have accuracy to two decimal places).

a) Find the x-intercepts of the graph and the y-intercept.

b) P behaves like what function for large $|x|$?

c) Find the turning points of P.

d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.

e) Use the graph to determine where the function is increasing.

a) x-intercepts:

$x = 3, -4, 5$

y-int: $y = P(0) = -(-3)\cdot 4\cdot(-5) = -60$

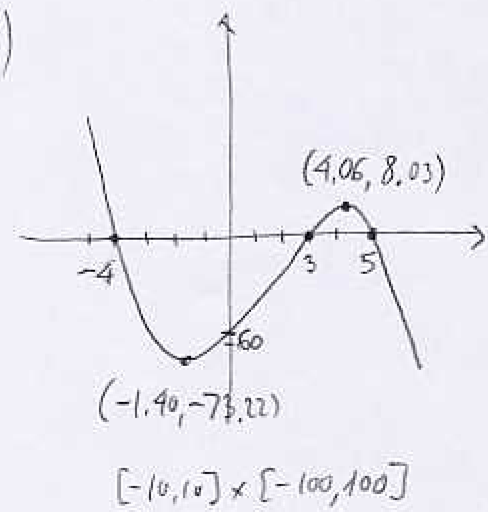
b) $P(x) = -(x^2 + \_\_)(x-5)$

$= -(x^3 + \_\_\_\_)$

$= -x^3 - \underline{\text{lower powers}}$

Behaves like $-x^3$

d)



c) Turning points:

$x = -1.40,\ y = -73.22$

$x = 4.06 \quad y = 8.03$

e) Increasing on $(-1.4, 4.06)$

6. (2pts) Write a formula for a polynomial of degree 3 whose zeroes are -3 (multiplicity 2) and 4 (multiplicity 1).

$$\underbrace{P(x) = (x+3)^2 (x-4)}_{\text{has degree } 3}$$

7. (11pts) Consider the rational function $Q(x) = \dfrac{3x+5}{x^2-3x-10}$.
Answer the following (decimal answers should have accuracy to two decimal places).
a) Find the domain of the function and where the vertical asymptotes are.
b) Find the x-intercepts of the graph and the y-intercept.
c) Find the horizontal asymptote, if any.
d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.
e) Find the intervals where the function is increasing.

a) $x^2 - 3x - 10 = 0$

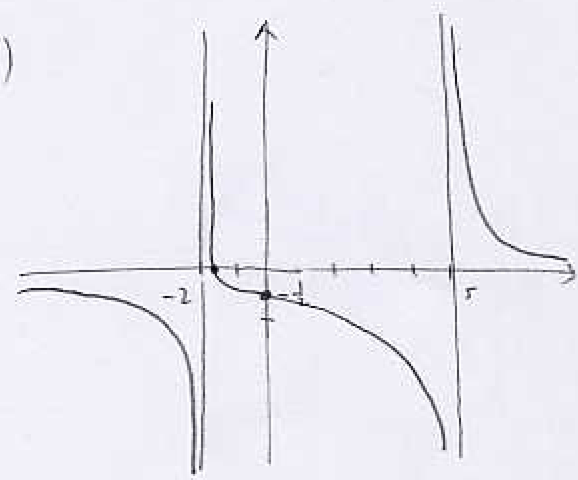
$(x-5)(x+2) = 0$

$x = 5, -2$

$D = \{x \mid x \neq 5, x \neq -2\}$

Asymp: $x = 5, x = -2$

d)



b) $3x + 5 = 0$

$x = -\frac{5}{3}$ x-int

$Q(0) = \frac{5}{-10} = -\frac{1}{2}$ y-int

e) Not increasing anywhere

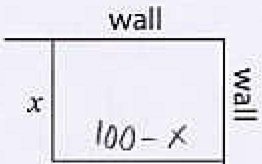
c) deg P < deg Q so
horizontal asymp. is $y = 0$

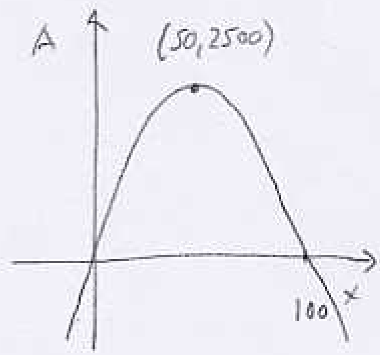
8. (8pts) Shannon has 100ft of fencing to enclose a rectangular play pen. Two sides of the pen are walls (see picture) and fence is used for the remaining two sides.

a) Express the area A of the play pen as a function of the width x.

b) Draw an accurate graph of the function $A(x)$.

c) For what x is the area the largest? What is the maximum area?





a) $A = x(100-x)$

$= -x^2 + 100x$

b) $x(100-x) = 0$

$x = 0, 100$ x-int.

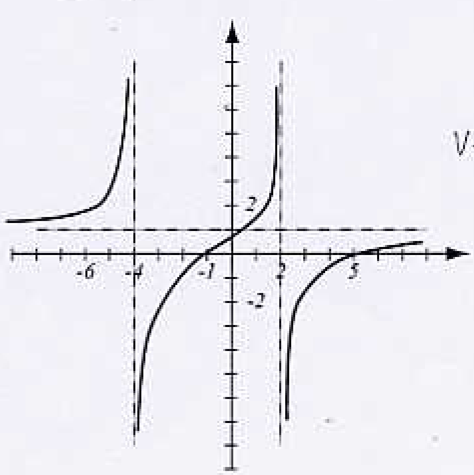
vertex: $x = -\frac{100}{2\cdot(-1)} = 50$

$y = 50(100-50) = 2500$

c) Area is largest when $x = 50$ ft

Max area is 2500 sq. ft

Bonus (5pts) Find the formula for a rational function whose graph is shown. (Hint: what will give you the correct vertical asymptotes? The correct x-intercepts?)



x-int: $-1, 5 \Rightarrow$ has $(x-(-1)), (x+5)$ in num.

vert. asymptotes: $\left.\begin{array}{l} x=-4 \\ x=2 \end{array}\right\} \Rightarrow$ must have $(x-(-4)), (x-2)$ in denominator

$$f(x) = \frac{(x+1)(x-5)}{(x+4)(x-2)}$$