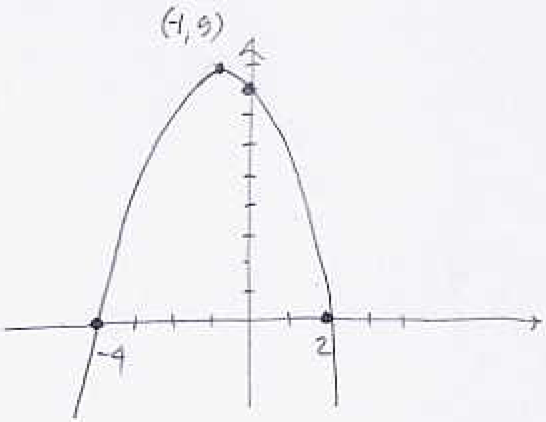
Spring '06/MAT 140/Exam 3a **Name:** Solution *Show all your work.*

1. (8pts) The quadratic function $f(x) = -x^2 - 2x + 8$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.
d) What is the range of the function?



a) $-x^2 - 2x + 8 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, 2$

b) $x = -\frac{b}{2a} = -\frac{-2}{2(-1)} = -1$

$y = -(-1)^2 - 2(-1) + 8 = 9$

c) Range is $(-\infty, 9]$

2. (2pts) The table gives values of f and g for some x's. Find $(g \circ f)(2)$ and $(f \circ f)(3)$.

x	1	2	3
$f(x)$	1	2	1
$g(x)$	3	1	2

$(g \circ f)(2) = g(f(2)) = g(2) = 1$

$(f \circ f)(3) = f(f(3)) = f(1) = 1$

3. (5pts) Let $f(x) = x^2 + 5$ and $g(x) = \sqrt{x - 7}$. Find the following composites (simplify if possible):

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x-7})$

$= (\sqrt{x-7})^2 + 5$

$= x - 7 + 5$

$= x - 2$

$(g \circ g)(x) = g(g(x))$

$= g(\sqrt{x-7})$

$= \sqrt{\sqrt{x-7} - 7}$

4. (3pts) Let $h(x) = |3x + 5|$. Break up this function into a composite of two functions f and g. That is, find f and g so that $h(x) = (f \circ g)(x)$.

1) find $3x+5$ $\qquad$ $g(x) = 3x+5$

2) take abs. value of result $\qquad$ $f(x) = |x|$

5. (11pts) Consider the polynomial $P(x) = (x + 1)(x - 4)(x - 6)$. Answer the following (decimal answers should have accuracy to two decimal places).

a) Find the x-intercepts of the graph and the y-intercept.

b) P behaves like what function for large $|x|$?

c) Find the turning points of P.

d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.

e) Use the graph to determine where the function is decreasing.

a) x-int:

$(x+1)(x-4)(x-6) = 0$

$x = -1, 4, 6$

y-int:

$P(0) = 1 \cdot (-4)(-6) = 24$

b) $P(x) = (x^2 - \ \ )(x-6)$

$= x^3 + \underline{\text{lower powers}}$

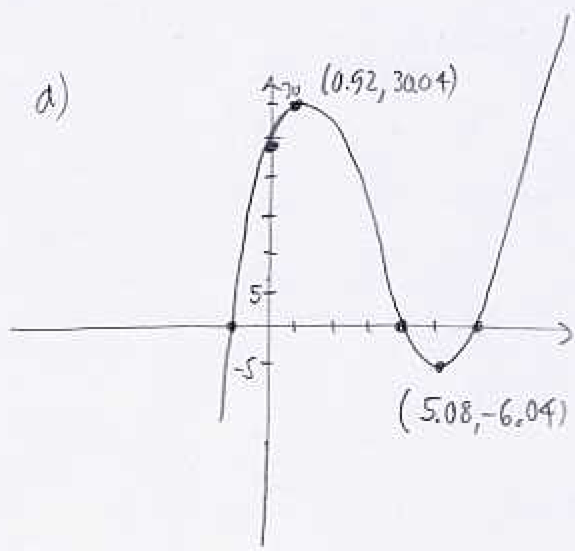
Behaves like x^3

c) Turning pts:

$x = 0.92,\ y = 30.04$

$x = 5.08,\ y = -6.04$

d)



e) decreasing on $(0.92, 5.08)$

6. (2pts) Write a formula for a polynomial of degree 4 whose zeroes are 1 (multiplicity 3) and 7 (multiplicity 1).

$P(x) = \underbrace{(x-1)^3(x-7)}_{\text{has degree } 4}$

7. (11pts) Consider the rational function $Q(x) = \dfrac{x^2 - 3x - 10}{3x + 5}$.

Answer the following (decimal answers should have accuracy to two decimal places).

a) Find the domain of the function and where the vertical asymptotes are.

b) Find the x-intercepts of the graph and the y-intercept.

c) Find the horizontal asymptote, if any.

d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.

e) Find the intervals where the function is decreasing.

a) $3x+5=0$

$x = -\frac{5}{3}$

$D = \{x \mid x \neq -\frac{5}{3}\}$

Vert. asymp at $x = -\frac{5}{3}$

b) $x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

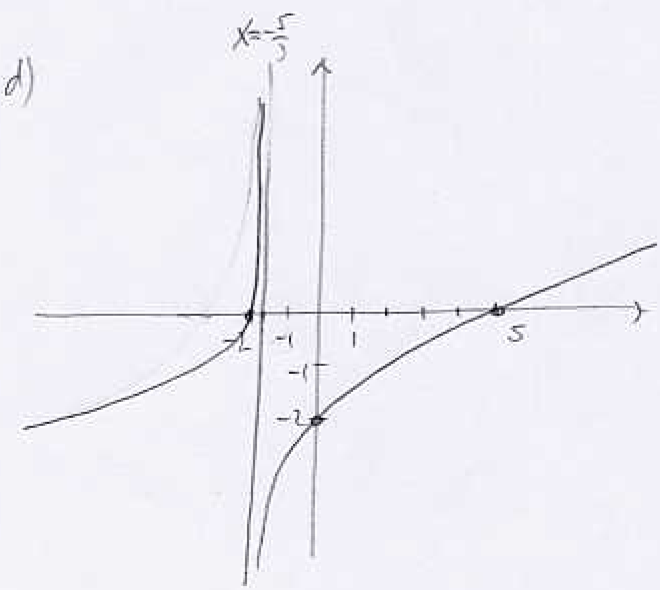
$x = -2, 5$ x-int

y-int

$Q(0) = \frac{-10}{5} = -2$

c) deg num > deg denom.
so no horizontal asymptote

d)



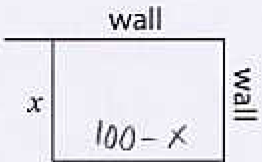
e) not decreasing anywhere

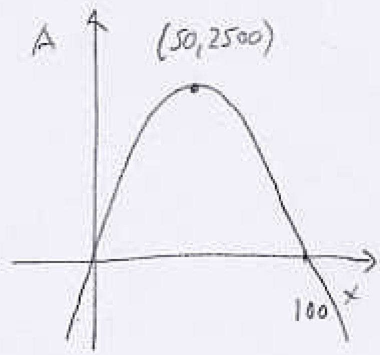
8. (8pts) Shannon has 100ft of fencing to enclose a rectangular play pen. Two sides of the pen are walls (see picture) and fence is used for the remaining two sides.

a) Express the area A of the play pen as a function of the width x.

b) Draw an accurate graph of the function $A(x)$.

c) For what x is the area the largest? What is the maximum area?





a) $A = x(100-x)$

$= -x^2 + 100x$

b) $x(100-x) = 0$

$x = 0, 100$ x-int.

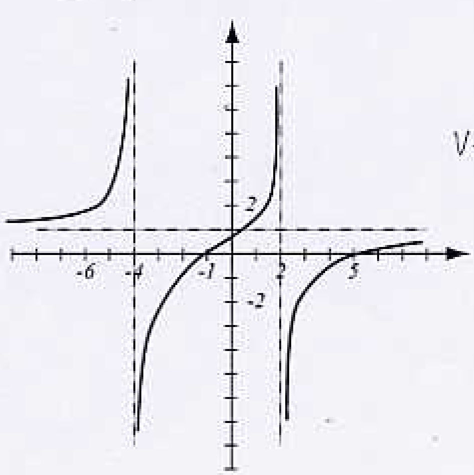
vertex: $x = -\dfrac{100}{2\cdot(-1)} = 50$

$y = 50(100-50) = 2500$

c) Area is largest when $x = 50$ ft

Max area is 2500 sq. ft

Bonus (5pts) Find the formula for a rational function whose graph is shown. (Hint: what will give you the correct vertical asymptotes? The correct x-intercepts?)



x-int: $-1, 5 \Rightarrow$ has $(x-(-1))$, $(x+5)$ in num.

vert. asymptotes: $\left.\begin{array}{l} x = -4 \\ x = 2 \end{array}\right\} \Rightarrow$ must have $(x-(-4))$, $(x-2)$ in denominator

$$f(x) = \frac{(x+1)(x-5)}{(x+4)(x-2)}$$