Spring '06/MAT 140/Exam 2b   Name: *Solution*   *Show all your work.*

1. (4pts) The following are graphs of basic functions. Write the equation of the graph under each one.

$y = \sqrt{x}$ $y = \dfrac{1}{x}$ $y = x^3$ $y = x^2$
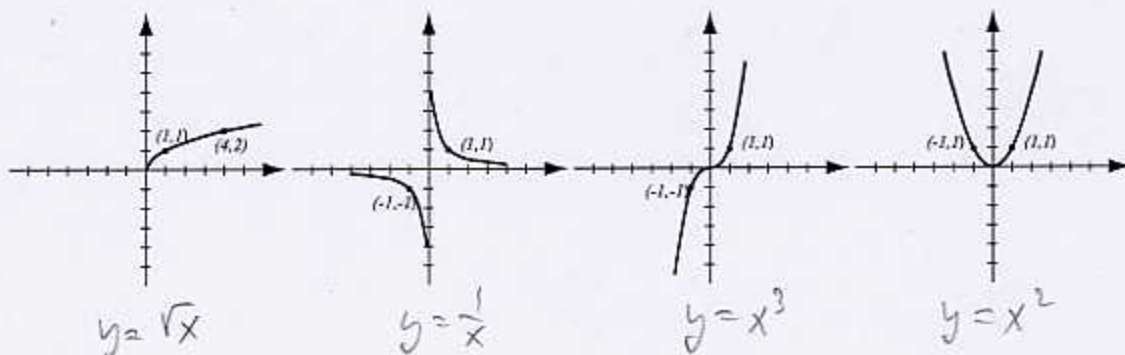
2. (4pts) Find the domain of the function $f(x) = \sqrt{3x - 7}$.

Must have $3x - 7 \geq 0$

$3x \geq 7$ Domain $= \left[\frac{7}{3}, \infty\right)$

$x \geq \frac{7}{3}$

3. (5pts) Find the equation of the line that passes through $(-2, 3)$ and is perpendicular to the line $3x + 2y = 6$. Draw both lines in the same coordinate system.

$3x + 2y = 6$

$2y = -3x + 6$

$y = -\frac{3}{2}x + 3$

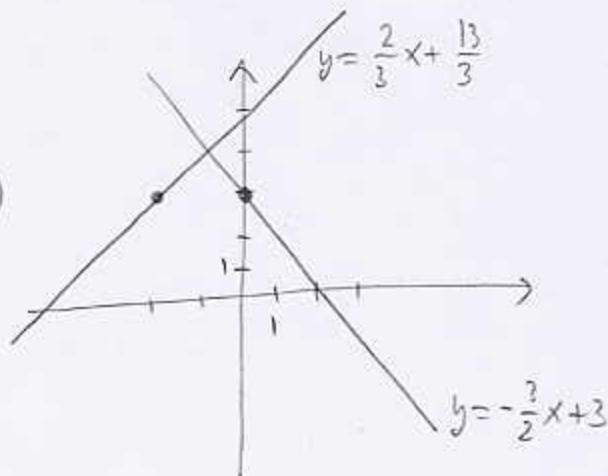slope is $-\frac{3}{2}$,

slope of perp. line is $\frac{2}{3}$

Perpendicular line:

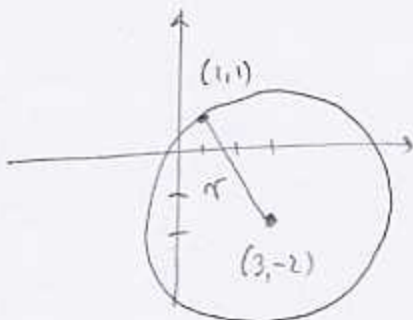$y - 3 = \frac{2}{3}(x - (-2))$

$y = \frac{2}{3}x + \frac{4}{3} + 3$

$y = \frac{2}{3}x + \frac{13}{3}$

$y = \frac{2}{3}x + \frac{13}{3}$

$y = -\frac{3}{2}x + 3$

4. (5pts) Find the equation of the circle whose center is $(3, -2)$ that contains the point $(1, 1)$. Draw the circle.

$$r = d\left((3, -2), (1, 1)\right)$$

$$= \sqrt{(3-1)^2 + (-2-1)^2} = \sqrt{4+9} = \sqrt{13}$$

Equation of circle:

$$(x-3)^2 + (y-(-2))^2 = \sqrt{13}^2$$

$$(x-3)^2 + (y+2)^2 = 13$$

5. (10pts) Use the graph of the function f at right to answer the following questions.

a) What is the domain of f? $(-6, 4]$

b) What is the range of f? $(-6, 4]$

c) Find $f(4)$ and $f(2)$. $\quad f(4) = -2$
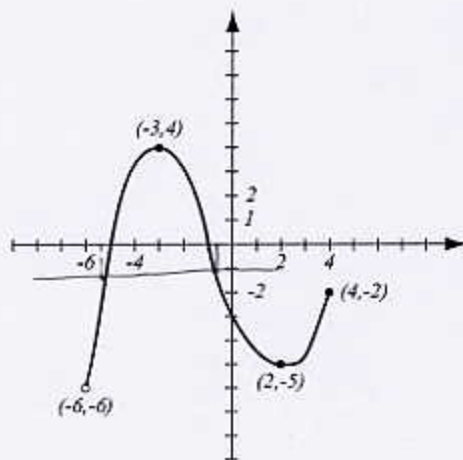$\qquad\qquad\qquad\qquad f(2) = -5$

d) List the x-intercepts of the graph.
$\qquad -5$ and -1

e) Where does f have a local maximum? What is its value? At $x = -3$ with value $y = 4$

f) What are the solutions of the equation $f(x) = -1$? $\quad x = -0.8$ and $x = -5.4$

g) For which x is $f(x) > 0$?

$\qquad$ for $x \in (-5, -1)$

6. (5pts) A bank offers a 30-year loan with a certain fixed interest rate. Under the terms of such a loan, one borrower secured a 30-year loan of \$110,000 with a montly payment of \$700.

a) Write the function that relates the monthly payment y to the amount borrowed x on such a loan. (y is proportional to x).

b) What is the monthly payment of a borrower who gets a \$170,000 loan?

a) $\quad y = mx$

$\quad 700 = m \cdot 110000$

$m = \dfrac{700}{110000} = \dfrac{7}{1100} = 0.006363\ldots$

$y = 0.00636363\ldots x$

b) $y = 0.006363\ldots \cdot 170,000$

$= 1081.82$

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7. (7pts) The function $f(x) = x^4 - 6x^2 + 5$ is given.

a) Determine algebraically whether this function is even, odd or neither.

b) Sketch the graph of f on paper. Why does your picture support what you found in a)?

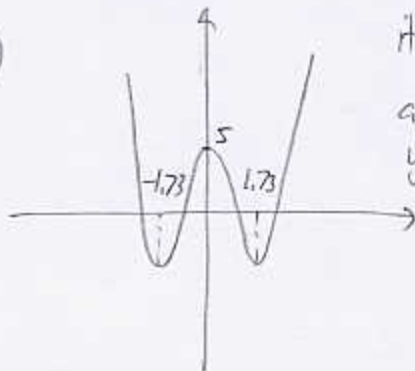c) List the intervals where f is increasing or decreasing. Accuracy: 2 decimal points.

a) $f(-x) = (-x)^4 - 6(-x)^2 + 5$

$\qquad = x^4 - 6x^2 + 5$

$\qquad = f(x)$

it is even

b)

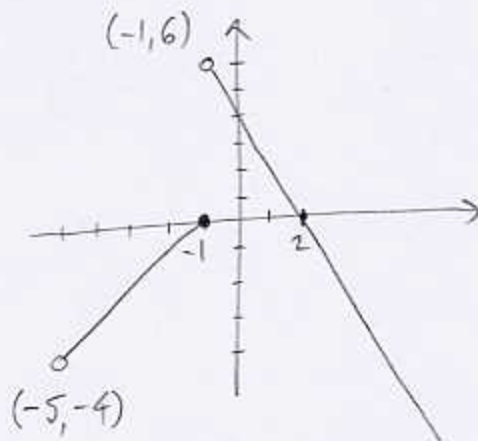It is symmetric about the y-axis

c) increasing on : $(-1.73, 5)$ and $(1.73, \infty)$
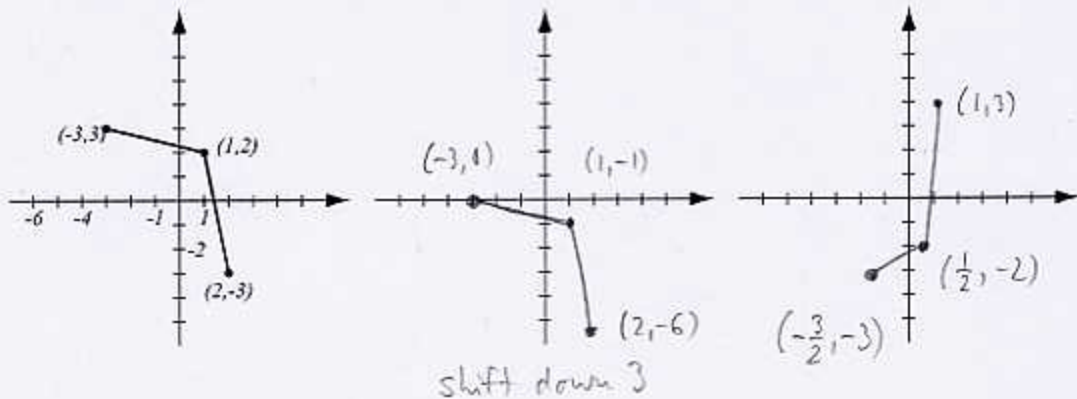
decreasing on : $(-\infty, -1.73)$ and $(0, 1.73)$

8. (5pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x+1, & \text{if } -5 < x \le -1 \\ -2x+4, & \text{if } -1 < x. \end{cases}$$

x	$x+1$
-5	-4
-1	0

x	$-2x+4$
-1	6
2	0

$(-1,6)$

$(-5,-4)$

9. (5pts) The graph of the function f is given below. On separate graphs, sketch the graphs of the functions $f(x) - 3$ and $-f(2x)$. Label all the relevant points.

$(-3,3)$ $(1,2)$ $(2,-3)$

$(-3,1)$ $(1,-1)$ $(2,-6)$

shift down 3

$(1,3)$ $(\frac{1}{2},-2)$ $(-\frac{3}{2},-3)$

stretch horizontally by factor $\frac{1}{2}$, reflect in x-axis

Bonus. (5pts) The following is an equation of a circle. Bring the equation into standard form in order to find its center and radius.

$$x^2 + 10x + y^2 - 4y + 15 = 0 \qquad \Big| + 5^2 + 2^2$$

$$x^2 + 2\cdot x \cdot 5 + 5^2 + y^2 - 2\cdot y \cdot 2 + 2^2 + 15 = 29 \qquad | -15$$

$$(x+5)^2 + (y-2)^2 = 14$$

$$(x-(-5))^2 + (y-2)^2 = (\sqrt{14})^2$$

Center $(-5,2)$ radius $\sqrt{14}$.