Spring '06/MAT 140/Exam 1b **Name:** Solution *Show all your work.*

1. (3pts) Identify each of the following numbers as integer, rational or irrational.

$\sqrt{11}$ irrational

4.263145297... (nonrepeating digits) irrational

$\frac{3}{4}$ rational

2. (2pts) Convert to or from scientific notation:

$5.76 \times 10^{-4} =$ 0.000576

$34,492,135 =$ 3.4492135×10^{7}

3. (4pts) Simplify and write the answer so all exponents are positive:

$$\frac{(2x)^3(x^{-4}y^4)^2}{(10x)^2y^{-7}} = \frac{8x^3x^{-8}y^8}{100x^2y^{-7}} = \frac{2x^{-5}y^8}{25x^2y^{-7}} = \frac{2x^{-5-2}y^{8-(-7)}}{25} = \frac{2x^{-7}y^{15}}{25} = \frac{2y^{15}}{25x^7}$$

4. (4pts) Simplify.

$$\frac{x+4}{x^2+4x-21} - \frac{x}{x^2-9} = \frac{x+4}{(x-3)(x+7)} - \frac{x}{(x-3)(x+3)}$$

$$= \frac{(x+4)(x+3) - x(x+7)}{(x-3)(x+7)(x+3)}$$

$$= \frac{x^2+7x+12-(x^2+7x)}{(x-3)(x+7)(x+3)}$$

$$= \frac{12}{(x-3)(x+7)(x+3)}$$

5. (4pts) Use a known formula to factor:

$16x^2 - 25 =$ $(4x)^2 - 5^2 = (4x-5)(4x+5)$

$x^3 + 27 =$ $x^3 + 3^3 = (x+3)(x^2 - 3x + 9)$

6. (3pts) Which of the points $A = (-4, 5)$ and $B = (6, -2)$ is closer to the origin? Justify your answer by a computation.

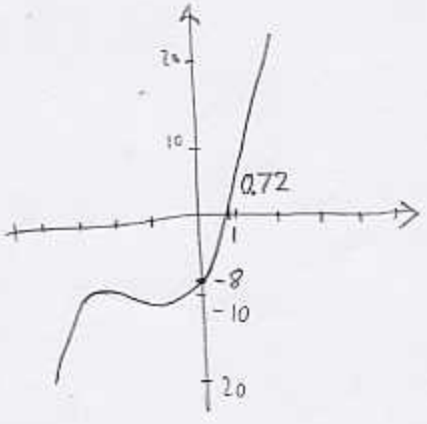
$d(A,0) = \sqrt{(-4-0)^2 + (5-0)^2} = \sqrt{16+25} = \sqrt{41}$

$d(B,0) = \sqrt{(6-0)^2 + (-2-0)^2} = \sqrt{36+4} = \sqrt{40}$ smaller

B is closer to the origin

7. (5pts) The equation $y = x^3 + 5x^2 + 7x - 8$ is given.
a) Use your calculator to help you sketch the graph (yes, on paper!). Make sure all the features of the graph are visible and indicate your viewing window.
b) Find the all the x-intercepts and the y-intercept to two decimal places.

a)



b) y-int (put in x=0)

$y = -8$

x-int: 0.72

8. (15pts) Solve the equations for x.

$cx - dx + 2d = d^2x + 5c - d$

$cx - dx - d^2x = 5c - d - 2d$

$x(c - d - d^2) = 5c - 3d$

$x = \dfrac{5c - 3d}{c - d - d^2}$

$x^2 - 4x = x + 36$

$x^2 - 5x - 36 = 0$

$(x - 9)(x + 4) = 0$

$x = 9, -4$

$|5x - 2| = 14$

$5x - 2 = 14$ or $5x - 2 = -14$

$5x = 16$ $\quad 5x = -12$

$x = \dfrac{16}{5}$ $\quad x = -\dfrac{12}{5}$

$x + 2 = \sqrt{x + 14}$ $\quad$ ( )^2

$x^2 + 4x + 4 = x + 14$

$x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

$x = -5, 2$

$2 + 2 \stackrel{?}{=} \sqrt{16}$ yes

$-5 + 2 \stackrel{?}{=} \sqrt{9}$ no

only $x = 2$ is a solution

9. (4pts) Mary-Kate can sign a stack of promotional photos in 40 minutes, while Ashley needs 30 minutes to accomplish the same job. How long does it take them to sign a stack of photos if they work together? (Of course, each photo gets signed only once.)

T = time it takes them together

In 1 minute, Mary Kate does $\frac{1}{40}$ of job

Ashley " $\frac{1}{30}$ " "

together they do $\frac{1}{T}$ " "

$\dfrac{1}{40} + \dfrac{1}{30} = \dfrac{1}{T}$

$\dfrac{3 + 4}{120} = \dfrac{1}{T}$

$\dfrac{7}{120} = \dfrac{1}{T}$

$T = \dfrac{120}{7}$ min

$= 17.14$ min

10. (6pts) The area of a rectangle is 40 square in. If the length is 4in greater than the width, what are the dimensions of the rectangle?

$w(w+4) = 40$

$w^2 + 4w - 40 = 0$

$w = \frac{-4 \pm \sqrt{16 - 4 \cdot (-40)}}{2}$

$= \frac{-4 \pm \sqrt{176}}{2}$

$= \frac{-4 \pm \sqrt{16 \cdot 11}}{2}$

$= \frac{-4 \pm 4\sqrt{11}}{2}$

$= -2 \pm 2\sqrt{11}$

$w = 4.63, -8.63$ ↑ negative, so can't use

rectangle is 4.63×8.63

Bonus. (5pts) Two cars drive along the same highway and start in the same spot. The faster car drives 10 mph faster than the slower car. If the faster car starts 10 minutes after the slower car, and catches up after 20 minutes of driving, how fast was each car going?

v = velocity of slower car — travels 30 min ($\frac{1}{2}$ hr)

$v+10$ = velocity of faster car — travels 20 min ($\frac{1}{3}$ hr)

Distance covered is same: ($s = v \cdot t$)

faster car slower car

$v \cdot \frac{1}{2} = \frac{1}{3}(v+10)$ 1.6

$3v = 2(v+10)$

$3v = 2v + 20$

$v = 20$

Slower car: 20 mph

Faster car: 30 mph