1. (11pts) Use formulas to expand:

a) $(3x - 2)(3x + 2) =$

b) $(4x - 5)^2 =$

c) $(x^2 + 7y)^2 =$

d) $(x + 4)^3 =$

2. (8pts) Factor the following. Use either a known formula or a factoring method.

a) $x^2 - 5x - 14 =$

b) $2x^2 - 9x - 18 =$

c) $x^3 - 125 =$

3. (3pts) Verify the formula for the difference of cubes by multiplying out:

$(x - a)^3 =$

4. (8pts) Simplify.

a) $\dfrac{x - 5}{x^2 - 2x - 8} + \dfrac{2x}{x^2 - 16} =$

b) $\dfrac{x + \dfrac{2x - 6}{x - 1}}{4 + \dfrac{16}{x - 1}} =$

5. (2pts) Solve the equation: $-4(x+3) = 2x + 1$

6. (3pts) Solve for x: $ax - a^2 = 2ab - bx + b^2$

7. (6pts) A quadrangle is a parallelogram if the midpoints of the opposite vertices are the same.
a) Use the above fact to see whether the quadrangle with vertices $A = (-3, 1)$, $B = (1, 4)$, $C = (0, 8)$, $D = (2, 3)$ is a parallelogram.
b) Find the perimeter of the quadrangle in a).

8. (7pts) Solve the following equations:

a) $x^2 = 2x + 24$
b) $x^2 - x = 6x - 20$

9. (6pts) A 2-lb steak is to be divided among Jeff, Sybill and Christine according to how many buttons there are on their clothing (go figure!). Jeff is wearing 5 buttons, Sybill 6 and Christine 8 buttons, hence Jeff gets $\frac{5}{6}$ of what Sybill gets, and Sybill gets $\frac{6}{8}$ of what Christine gets. How many pounds of steak does each of them receive?

10. (6pts) The height of a triangle is 3ft more than the base. What are the base and height if the area of the triangle is 40 square feet?

11. (8pts) Put the following expressions into standard form $a + bi$:

a) $(1 + i)(2i - 1) + 3i(i - 1) =$

b) $\dfrac{1 + i}{5 - 2i} =$

c) (justify also) $i^{42} =$

Solve the equations algebraically:

12. (5pts) $x^4 + 2x^2 - 35 = 0$

13. (4pts) $|2x - 3| = 5$

14. (7pts) Erin and Claudia bike to the same grocery store. It takes Claudia 15 minutes and Erin 20 minutes to ride to the store, since Erin lives 2 miles farther away than Claudia. Erin's bike speed is 4mph more than Claudia's.
a) What are the women's bike speeds?
b) How far is the store from Erin's house?
(Hint: convert time to hours.)

**15.** (6pts) How much water needs to be added to 3 liters of a 20% solution of muriatic acid in order to get a 15% solution?

16. (5pts) Let f be given by $f(x) = (x^2 + x - 1)x$. Find the following values for this function: $f(-1)$, $f(3u)$, $f(x+4)$. (Simplify where possible).

17. (4pts) Find the domain of $f(x) = \dfrac{\sqrt{5-x}}{x-2}$.

18. (9pts) Use the graph of the function f
at right to answer the following questions.
a) What is the domain of f?
b) Find $f(4)$ and $f(-1)$.
c) List the x-intercepts of the graph.
d) Where is the function decreasing?
e) What are the solutions of the equation
$f(x) = -4$?
f) Where is $f(x) > 0$?
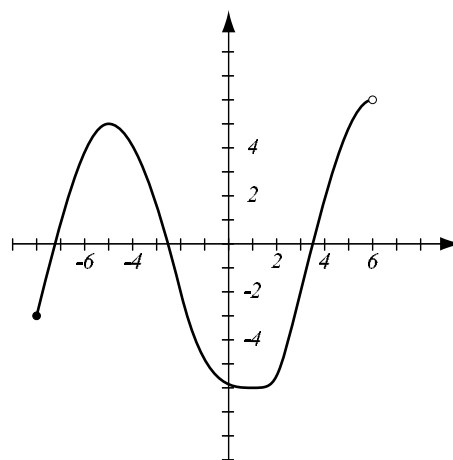
19. (8pts) The function $f(x) = x^4 - 4x^2 - 3x + 4$ is given.
a) Sketch the graph of f on paper.
b) List the numbers where f has a local minimum or maximum. What are the local minima (i.e. the y-values)? Accuracy: 3 decimal points.
c) List the intervals where f is increasing.
d) What is the range of this function?

20. (4pts) The function $f(x) = x^5 - 4x^3 + 7x$ is given.
a) Determine algebraically whether this function is even, odd or neither.
b) Graph the function on paper. Does the graph support your conclusion from a) and why?

21. (6pts) A horse farm buys a pick-up truck for $24,700. After two years the value of the truck is estimated to be $18,300.
a) Assuming the truck is depreciated using a linear function, write the function that expresses the value of the truck as a function of its age.
b) When does the truck have value $5,000?

22. (6pts) The amount of money that a bank will allow you to borrow mainly depends on the interest rate and your annual income. The data in the table represent a sample of loans L that were given to people with income levels I at interest rate 7.5% for 30 years.
a) Draw the scatterplot of the data on paper. Does the relationship look linear?
b) Use the calculator to find the "line of best fit" to the data. Draw the line on paper.
c) Interpret the slope of the line of best fit.
d) What loan amount would an individual earning $37,500 qualify for?

I	L
20,000	60,500
25,000	74,500
30,000	90,400
35,000	104,300
40,000	118,200
45,000	134,100
50,000	152,000
55,000	163,900

**23.** (6pts) The function f is given below.

a) Sketch the graph of f on paper.

b) Find the domain and range of f.

$$f(x) = \begin{cases} -2x - 3, & \text{if } -4 \le x \le 3 \\ x - 1, & \text{if } 3 < x < 7. \end{cases}$$

24. (5pts) Use the basic graph of $y = \frac{1}{x}$ and transformations to help you sketch the graph of $y = \frac{2}{x+3}$. Explain how you transform the original graph and what the asymptotes of the new graph are.

25. (7pts) The graph of the function f is given below. On three separate graphs, sketch the graphs of the functions $f(x) + 2$, $f(2x)$ and $-f(x - 2)$. Label all the relevant points.
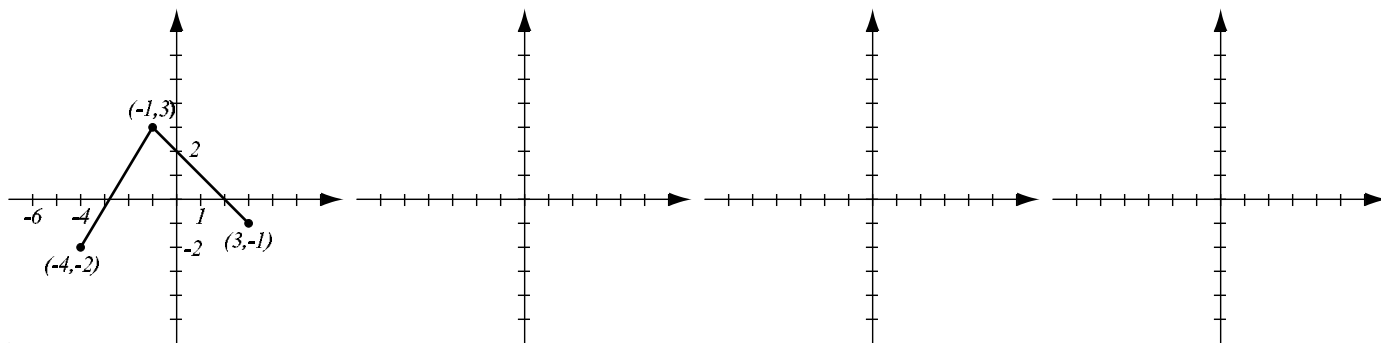
26. (4pts) The table below indicates the values of $f(x)$ and $g(x)$ for certain numbers. Find the requested composites at right.

x	-2	1	4	7	9
f(x)	9	-2	1	7	4
g(x)	4	9	7	1	-2

$(f \circ g)(4) =$

$(g \circ f)(7) =$

$(f \circ f)(-2) =$

$(g \circ g)(1) =$

27. (8pts) Let $f(x) = \sqrt{x+7}$ and $g(x) = x^2 + x - 1$. Find the following composites and simplify where possible:
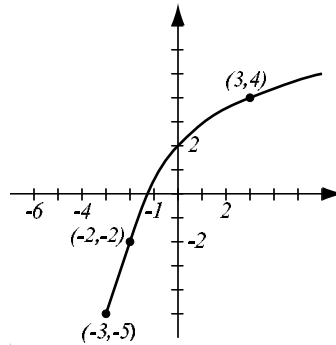
$(f \circ g)(x) =$

$(g \circ f)(x) =$

$(g \circ g)(x) =$

28. (4pts) Find functions f and g so that $f \circ g = H$, if $H(x) = \dfrac{4}{x+3}$. Find two different solutions to this problem, neither of which is the "stupid" one.

29. (4pts) The graph of a
function f is given. Use it to
find the graph of f^{-1}, label-
ing the relevant points.

30. (5pts) Find the inverse of $g(x) = \dfrac{2x - 3}{5x + 2}$ and the range of g^{-1}.

31. (5pts) Find the inverse of $f(x) = x^2 - 4$, $x \le 0$, and the range of f.

32. (4pts) Evaluate without using the calculator:

$\log_5 125 =$ $\log_8 \frac{1}{64} =$ $\log_4 \sqrt{2} =$ $\log_a \sqrt[4]{a^3} =$

33. (6pts) Solve the equations:

$\log_2(2x + 5) = 4$ $10^{3x-1} = 32$

34. (3pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 \dfrac{9^{2x-3}}{\sqrt{x + 7}} =$

35. (3pts) Write the following as a single logarithm. Simplify if possible.

$\frac{3}{2} \log x^{12} + 2 \log x^{11} =$

36. (2pts) Compute the following number using your calculator. Show how you obtained your number.

$\log_7 14 =$

37. (5pts) Solve the equation:

$\log_2(x - 3) + \log_2(x - 1) = 3$

38. (7pts) At an archaelogical dig, the remains of a person were found. Test indicated that the amount of carbon 14 in their body was 30% of the original amount. How long ago did this person die? (The half-life of carbon 14 is 5600 years.)

39. (8pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.
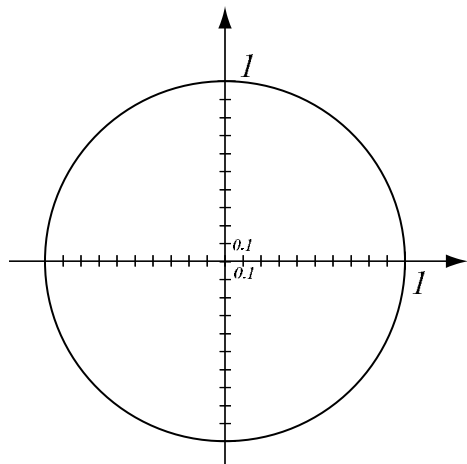
$$\sin 210° = \qquad\qquad \cot \frac{4\pi}{3} = \qquad\qquad \sec(-\pi) = \qquad\qquad \tan 150° =$$

40. (4pts)   Use the picture below to estimate $\sin \frac{7\pi}{8}$ and $\cos \frac{7\pi}{8}$. Then evaluate with a calculator and compare the results.

41. (5pts) If $\cos\theta = -\frac{\sqrt{5}}{7}$ and θ is in the third quadrant, find $\sin\theta$, $\cot\theta$, $\sec\theta$. Draw a picture.

42. (5pts) Draw two periods of the graph of $y = 3\cos(4x)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).

43. (5pts) A ship, offshore from a statue known to be 70ft tall, takes a sighting of the top of the statue. If the angle of elevation is $12°$, how far offshore is the ship?

44. (3pts) Use the unit circle to find the domain of $\cot\theta$.

45. (6pts) Without using the calculator, find the exact values of the following inverse trigonometric functions. Draw the unit circle and the appropriate angle under the expression.

$$\arcsin \frac{\sqrt{2}}{2} = \qquad\qquad \arccos\left(-\frac{\sqrt{3}}{2}\right) = \qquad\qquad \arctan \frac{1}{\sqrt{3}} =$$

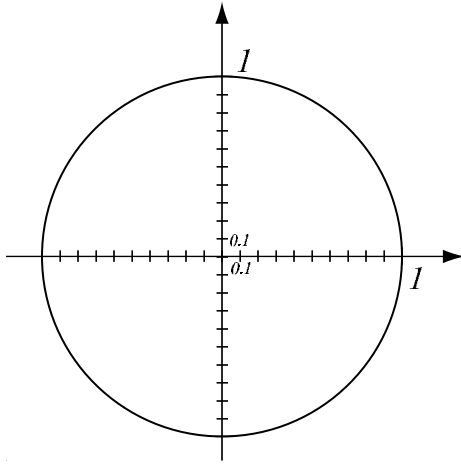**46.** (3pts) Use a picture to find the exact value below. Do not use the calculator.

$$\arccos\left(\cos\left(\frac{6\pi}{5}\right)\right) =$$

47. (4pts) Find the exact value of the expression below. Draw a picture and do not use the calculator.

$$\sin(\arctan(-4)) =$$

48. (4pts) Use the picture below to estimate (in degrees) arcsin(0.7) and arctan(−0.9). Then evaluate these numbers using a calculator and compare your answers.

49. (4pts) Find the exact value of $\cos 105°$. Do not use the calculator.

50. (4pts) Let $\cos\theta = \dfrac{1}{4}$, where $-\dfrac{\pi}{2} \le \theta \le 0$. Find $\sin\dfrac{\theta}{2}$.

51. (5pts) Show the identity: $\dfrac{1-\cos\theta}{1+\cos\theta} = (\csc\theta - \cot\theta)^2$.