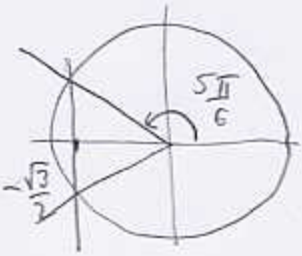
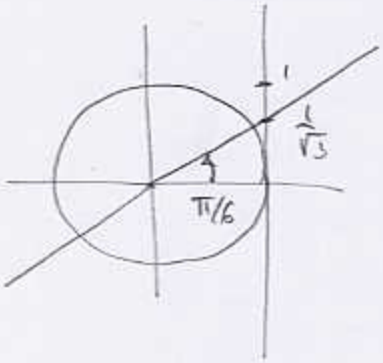
Fall '06/MAT 150/Worksheet 9 **Name:** Solution *Show all your work.*

1. (6pts) Without using the calculator, find the exact values of the following inverse trigonometric functions. Draw the unit circle and the appropriate angle under the expression.

$\arcsin \frac{\sqrt{2}}{2} = \frac{\pi}{4}$ $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ $\arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6}$

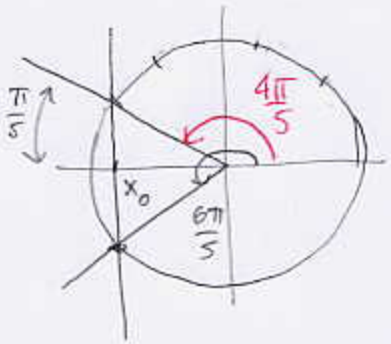




2. (3pts) Use a picture to find the exact value below. Do not use the calculator.

$\arccos\left(\cos\left(\frac{6\pi}{5}\right)\right) = \arccos x_0$

$x_0 = \frac{4\pi}{5}$



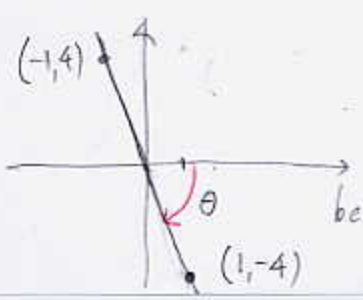
3. (4pts) Find the exact value of the expression below. Draw a picture and do not use the calculator.

$\sin(\arctan(-4)) =$ $\sin\theta = \frac{y}{r} = \frac{-4}{\sqrt{17}} = -\frac{4}{\sqrt{17}}$

Let $\theta = \arctan(-4)$

$\tan\theta = -4 \quad \theta$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$-4 = \frac{y}{x} = \frac{-4}{1} = \frac{4}{-1}$



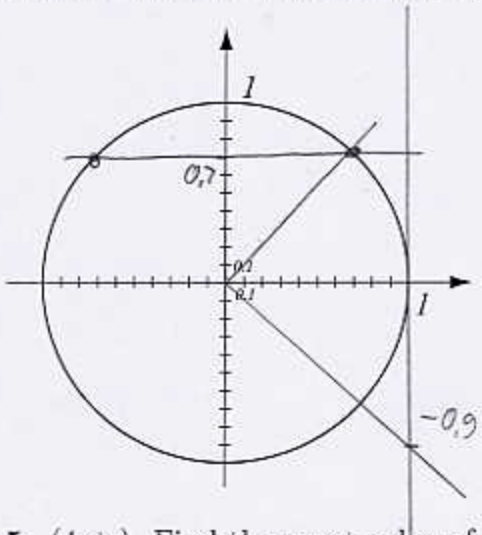
$r^2 = 1^2 + (-4)^2$

$r^2 = 17$

$r = \sqrt{17}$

because angle has to be in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

4. (4pts) Use the picture below to estimate (in degrees) arcsin(0.7) and arctan(−0.9). Then evaluate these numbers using a calculator and compare your answers.



	estimate	calculator
$\arcsin 0.7 \approx$	43°	44.427°
$\arctan(-0.9) \approx$	-41°	-41.987°

5. (4pts) Find the exact value of $\cos 105°$. Do not use the calculator.

$105° = 60° + 45°$

$\cos(60° + 45°) = \cos 60° \cos 45° - \sin 60° \sin 45°$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

6. (4pts) Let $\cos\theta = \frac{1}{4}$, where $-\frac{\pi}{2} \le \theta \le 0$. Find $\sin\frac{\theta}{2}$.

$\sin^2\frac{\theta}{2} = \frac{1 - \cos\theta}{2}$

$\sin^2\frac{\theta}{2} = \frac{1 - \frac{1}{4}}{2} \cdot \frac{4}{4} = \frac{3}{8}$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{3}{8}}$

Since $-\frac{\pi}{2} \le \theta \le 0$

$-\frac{\pi}{4} \le \frac{\theta}{2} \le 0$ so $\sin\frac{\theta}{2} < 0$

$\sin\frac{\theta}{2} = -\sqrt{\frac{3}{8}}$

7. (5pts) Show the identity: $\frac{1 - \cos\theta}{1 + \cos\theta} = (\csc\theta - \cot\theta)^2$.

$(\csc\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \frac{(1 - \cos\theta)^2}{\sin^2\theta} = \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta}$

$= \frac{(1 - \cos\theta)^{\cancel{2}}}{\cancel{(1 - \cos\theta)}(1 + \cos\theta)} = \frac{1 - \cos\theta}{1 + \cos\theta}$