Fall '06/MAT 140/Worksheet 7 **Name:** Solution *Show all your work.*

1. (4pts) Evaluate without using the calculator:

$\log_5 125 = 3$ $\log_8 \frac{1}{64} = -2$ $\log_4 \sqrt{2} = \frac{1}{4}$ $\log_a \sqrt[4]{a^3} = \frac{3}{4}$

$5^? = 125$ $8^? = \frac{1}{64} = \frac{1}{8^2}$ $4^? = \sqrt{2}$ $a^? = a^{\frac{3}{4}}$

$\sqrt{4} = 2$ $\sqrt{\sqrt{4}} = \sqrt{2}$ $(4^{\frac{1}{2}})^{\frac{1}{2}} = 4^{\frac{1}{4}}$

2. (6pts) Solve the equations:

$\log_2(2x+5) = 4$

$2^4 = 2x+5$

$11 = 2x$

$x = \frac{11}{2}$

$10^{3x-1} = 32 \mid \log$

$\log 10^{3x-1} = \log 32$

$3x-1 = \log 32$

$3x = \log 32 + 1$

$x = \frac{\log 32 + 1}{3} \approx 0.835$

3. (3pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 \frac{9^{2x-3}}{\sqrt{x+7}} = \log_3 9^{2x-3} - \log_3 (x+7)^{\frac{1}{2}} = (2x-3)\underbrace{\log_3 9}_{=2} - \frac{1}{2}\log_3(x+7)$

$= 4x - 6 - \frac{1}{2}\log_3(x+7)$

4. (3pts) Write the following as a single logarithm. Simplify if possible.

$\frac{3}{2}\log x^{12} + 2\log x^{11} = \log (x^{12})^{\frac{3}{2}} + \log (x^4)^2$

$= \log\left(x^{\cancel{12}^{6} \cdot \frac{3}{2}} \cdot (x^{11})^2\right)$

$= \log\left(x^{18} \cdot x^{22}\right)$

$= \log x^{40} = 40 \log x$

5. (2pts) Compute the following number using your calculator. Show how you obtained your number.

$\log_7 14 = \dfrac{\ln 14}{\ln 7} \approx 1.356$

6. (5pts) Solve the equation:

$\log_2(x-3) + \log_2(x-1) = 3$

$\log_2 (x-3)(x-1) = 3$

$x^2 - 4x + 3 = 2^3$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5, -1$ Only $x=5$ is a solution.

Test: $\log_2(2) + \log_2 4 = 3$

$1 + 2 = 3$ ok

$\log_2(-4) + \log_2(-2) = 3$

not defined

7. (7pts) At an archaelogical dig, the remains of a person were found. Test indicated that the amount of carbon 14 in their body was 30% of the original amount. How long ago did this person die? (The half-life of carbon 14 is 5600 years.)

$A(t) = A_0 e^{kt}$ $k = ?$

$\frac{1}{2} A_0 = A_0 e^{k \cdot 5600}$ $\mid \div A_0$

$\frac{1}{2} = e^{k \cdot 5600}$ $\mid \ln$

$\ln \frac{1}{2} = k \cdot 5600$

$\dfrac{\ln \frac{1}{2}}{5600} = k$

$k \approx -0.000124$

$0.3A_0 = A_0 e^{-0.000124 t}$ $\mid \div A_0$

$0.3 = e^{-0.000124 t}$ $\mid \ln$

$\ln 0.3 = -0.000124 t$

$t = \dfrac{\ln 0.3}{-0.000124} \approx 9727.007$ years

how long ago they died