Fall '06/MAT 150/Worksheet 6 Name: Solution *Show all your work.*

1. (4pts) The table below indicates the values of $f(x)$ and $g(x)$ for certain numbers. Find the requested composites at right.

x	-2	1	4	7	9
f(x)	9	-2	1	7	4
g(x)	4	9	7	1	-2

$(f \circ g)(4) = f(g(4)) = f(7) = 7$

$(g \circ f)(7) = g(f(7)) = g(7) = 1$

$(f \circ f)(-2) = f(f(-2)) = f(9) = 4$

$(g \circ g)(1) = g(g(1)) = g(9) = -2$

2. (8pts) Let $f(x) = \sqrt{x+7}$ and $g(x) = x^2 + x - 1$. Find the following composites and simplify where possible:

$(f \circ g)(x) = f(g(x)) = f(x^2 + x - 1) = \sqrt{x^2 + x - 1 + 7} = \sqrt{x^2 + x + 6}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+7}) = (\sqrt{x+7})^2 + \sqrt{x+7} - 1 = x + 7 + \sqrt{x+7} - 1$

$= x + 6 + \sqrt{x+7}$

$(g \circ g)(x) = g(g(x)) = g(x^2 + x - 1) = (x^2 + x - 1)^2 + x^2 + x - 1 - 1$

$= \overbrace{x^4 + x^3 - x^2 + x^3 + x^2 - x - x^2 - x + 1}^{(x^2+x-1)(x^2+x-1)} + x^2 + x - 2 = x^4 + 2x^3 - x - 1$

3. (4pts) Find functions f and g so that $f \circ g = H$, if $H(x) = \dfrac{4}{x+3}$. Find two different solutions to this problem, neither of which is the "stupid" one.

$f(g(x)) = \dfrac{4}{x+3}$

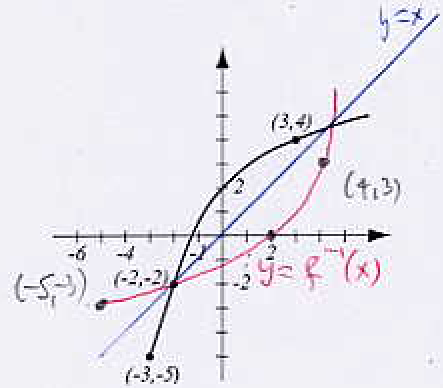
$g(x) = x + 3$

$f(x) = \dfrac{4}{x}$

$g(x) = \dfrac{1}{x+3}$

$f(x) = 4x$

4. (4pts) The graph of a function f is given. Use it to find the graph of f^{-1}, labeling the relevant points.



reflect in line $y=x$

5. (5pts) Find the inverse of $g(x) = \dfrac{2x-3}{5x+2}$ and the range of g^{-1}.

$$y = \frac{2x-3}{5x+2}$$

$$y(5x+2) = 2x-3$$

$$5xy + 2y = 2x-3$$

$$5xy - 2x = -2y-3$$

$$x(5y-2) = -2y-3$$

$$x = \frac{-2y-3}{5y-2} = \frac{2y+3}{2-5y}$$

$$g^{-1}(y) = \frac{2y+3}{2-5y}$$

Range of g^{-1} = $\underbrace{\text{domain of } f}$ = $\{x \mid x \neq -\frac{2}{5}\}$

$$5x+2=0$$

$$x = -\frac{2}{5}$$

6. (5pts) Find the inverse of $f(x) = x^2 - 4$, $x \leq 0$, and the range of f.

$$y = x^2 - 4$$

$$y + 4 = x^2$$

$$x = \pm\sqrt{y+4}$$

Since $x \leq 0$, we take the solution with $-$

$$x = -\sqrt{y+4}$$

Range of f = $\underbrace{\text{domain of } f^{-1}}$ = $[-4, \infty)$

must have $y+4 \geq 0$

$$y \geq -4$$