$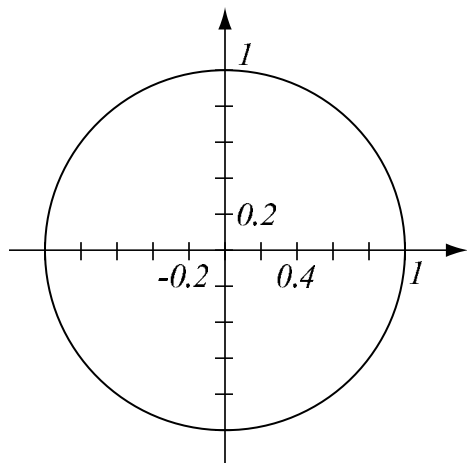$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \sin(2\theta) = 2\sin\theta\cos\theta \qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1 \qquad \cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad\qquad = 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta} \qquad \tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

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1. (8pts) Without using the calculator, find the exact values of the following expressions. Draw the unit circle and the appropriate angle under the expression.

$$\cos 30° = \qquad\qquad \tan\frac{5\pi}{2} = \qquad\qquad \arccos\frac{\sqrt{2}}{2} = \qquad\qquad \arcsin\left(-\frac{1}{2}\right) =$$

2. (4pts) Use the picture below to estimate $\tan 40°$ and $\arccos(-0.4)$ (in degrees). Then evaluate these numbers using a calculator and compare your answers.

3. (4pts) Draw two periods of the graph of $y = 2\cos(\pi x)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).

4. (4pts) Use an addition formula to find the exact value of $\cos 105°$.

5. (4pts) Use a double-angle formula to find the exact value of $\sin \dfrac{9\pi}{8}$.

6. (3pts) State the angles for which $\sec\theta$ is not defined. Explain. (Hint: looking at the unit circle and writing what $\sec\theta$ is in terms of x and y coordinates may help.)

7. (4pts) Show the identity: $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$.

8. (6pts) If $\tan \theta = \frac{2}{5}$ and θ is in the third quadrant, find $\sin(2\theta)$ and $\cos \dfrac{\theta}{2}$.

9. (4pts) Find the exact values of the expressions below. Draw a picture if helpful and do not use the calculator.

$\tan(\arctan 4.13) =$

$\arccos \left(\cos \left(\dfrac{5\pi}{4} \right) \right) =$

10. (4pts) Find all the solutions of the equation $2 \sin \theta - \sqrt{3} = 0$.

11. (5pts) Suppose you are headed toward a building 50 meters high. If the angle of elevation to the top of the building is $20°$, how far away from the building are you?

Bonus (5pts) Show that $\sin(\arctan u) = \dfrac{u}{\sqrt{u^2 + 1}}$.