Fall '06/MAT 150/Final Exam **Name:** Solution *Show all your work.*

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\qquad$ $\sin(2\theta) = 2\sin\theta\cos\theta$ $\qquad$ $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$ $\qquad$ $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\qquad$ $\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta}$

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

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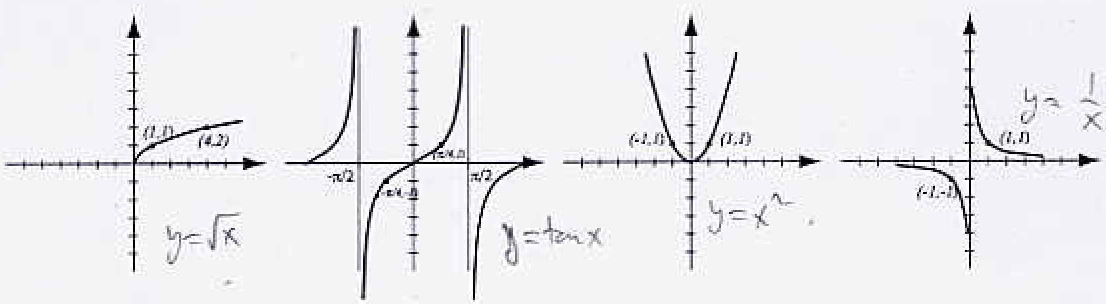
1. (4pts) Find the equation of the line that contains the points $(-2,4)$ and $(4,-3)$. Then find the equation of the line that is perpendicular to this one and passes through the origin.

$m = \frac{-3-4}{4-(-2)} = \frac{-7}{6} = -\frac{7}{6}$ $\qquad$ $y = -\frac{7}{6}x + \frac{5}{3}$ $\qquad$ Slope of perp. line is $-\frac{1}{-\frac{7}{6}} = \frac{6}{7}$

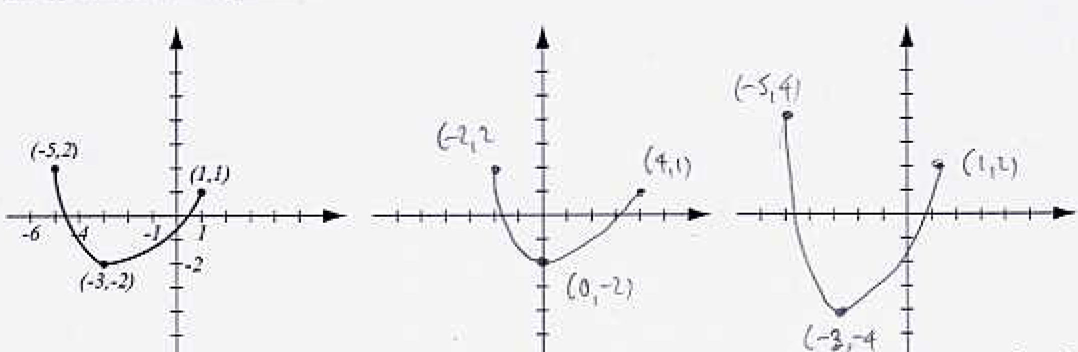
$y - 4 = -\frac{7}{6}(x-(-2))$ $\qquad$ $y - 0 = \frac{6}{7}(x-0)$

$y = -\frac{7}{6}x - \frac{7}{3} + 4$ $\qquad$ $y = \frac{6}{7}x$

2. (4pts) The following are graphs of basic functions that we have had in this course. Write the equation of the graph under each one.

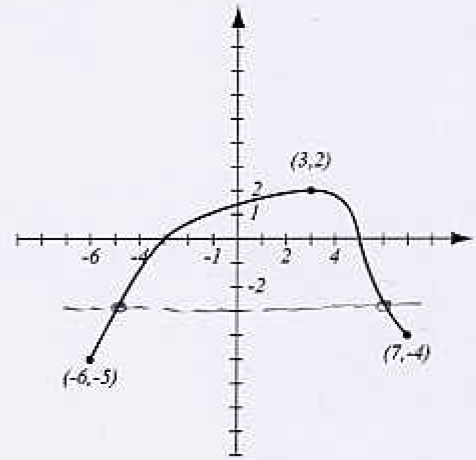


3. (4pts) The graph of $f(x)$ is drawn below. Find the graphs $f(x-3)$ and $2f(x)$ and label all the relevant points.



shift right 3 $\qquad$ stretch vertically, Factor=2

4. (6pts) Use the graph of the function f at right to answer the following questions.
a) What is $f(3)$?
b) What are the x-intercepts?
c) State the intervals of x's where $f(x) > 0$.
d) What are the solutions of the equation $f(x) = -3$?



a) $f(3) = 2$ c) $x \in (-3,5)$

b) $x = -3, 5$ d) $x = -5, 6$

5. (6pts) The quadratic function $f(x) = x^2 + 2x - 4$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) $x^2+2x-4=0$

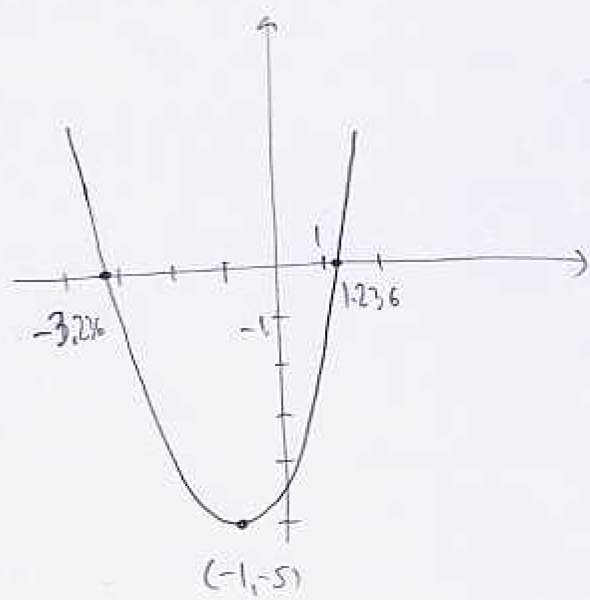
$$x = \frac{-2 \pm \sqrt{2^2-4\cdot(-4)}}{2}$$

$$= \frac{-2 \pm \sqrt{20}}{2}$$

$$= \frac{-2 \pm 2\sqrt{5}}{2}$$

$$= \frac{2(-1 \pm \sqrt{5})}{2}$$

$$= -1 \pm \sqrt{5} \approx \begin{matrix} -3.236 \\ 1.236 \end{matrix}$$



b) $x = -\frac{2}{2\cdot 1} = -1$

$y = (-1)^2 + 2(-1) - 4 = -5$

6. (8pts) The polynomial $f(x) = x^4 - 4x^2 - 3$ is given. Use your calculator to solve the following with accuracy 3 decimal points.
a) Find the x-intercepts and the y-intercept. Does f have the maximum number of x-intercepts that a fourth-degree polynomial can have?
b) Find the intervals of increase and decrease of this function.
c) What is the range of f?
d) Sketch the graph of the function on paper (large and clear — make Dad proud!).

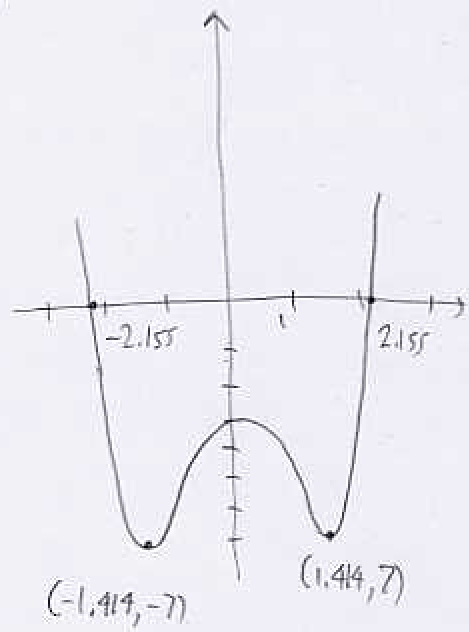
a) y-int: $f(0) = -3$
x-int: $2.155, -2.155$

No, it has two, where the maximal possible number is 4.

b) Increasing on $(-1.414, 0)$ and $(1.414, \infty)$
Decreasing on $(-\infty, -1.414)$ and $(0, 1.414)$

c) Range is $[-7, \infty)$

d)



7. (5pts) Solve the equation $x + 3 = \sqrt{4x + 18}$.

$(x+3)^2 = 4x+18$

$x^2+6x+9 = 4x+18$

$x^2+2x-9=0$

$x = \dfrac{-2 \pm \sqrt{4-4.1.(-9)}}{2}$

$= \dfrac{-2 \pm \sqrt{40}}{2} = \dfrac{-2 \pm 2\sqrt{10}}{2} = -1 \pm \sqrt{10}$

≈ 2.1622
-4.1622

Check: $-1+\sqrt{10}+3 \stackrel{?}{=} \sqrt{4(-1+\sqrt{10})+18}$

$5.162 \stackrel{?}{=} 5.162$ ... yes

$-1-\sqrt{10}+3 \stackrel{?}{=} \sqrt{4(-1-\sqrt{10})+18}$

$-4.162 \stackrel{?}{=}$ positive no

Only $-1+\sqrt{10}$ is a sol,

8. (3pts) Let $f(x) = x + 3$, $g(x) = x^2 + 3x - 1$ Find $(g \circ f)(x)$ and simplify.

$(g \circ f)(x) = g(f(x)) = g(x+3) = (x+3)^2 + 3(x+3) - 1$

$= x^2 + 6x + 9 + 3x + 9 - 1$

$= x^2 + 9x + 17$

9. (4pts) Evaluate without using the calculator:

$\log_4 16 = 2$ $\qquad$ $\log_3 \frac{1}{27} = -3$ $\qquad$ $\log_{49} 7 = \frac{1}{2}$ $\qquad$ $\log_a \sqrt[7]{a^3} = \frac{3}{7}$

$4^? = 16$ $\qquad$ $3^? = \frac{1}{27}$ $\qquad$ $49^? = 7$ $\qquad$ $a^? = a^{\frac{3}{7}}$

10. (3pts) Write as a difference of logarithms. Express powers as factors. Simplify if possible.

$\ln \frac{e^{x+2}}{(x-7)^4} = \ln e^{x+2} - \ln (x-7)^4 = x + 2 - 4\ln(x-7)$

11. (3pts) Solve the equation: $3^{2x-4} = 17$.

$3^{2x-4} = 17 \quad | \ln$

$\ln 3^{2x-4} = \ln 17$

$(2x-4)\ln 3 = \ln 17$

$2x - 4 = \frac{\ln 17}{\ln 3}$

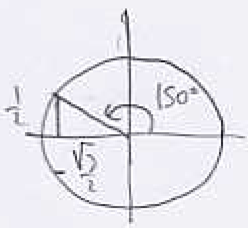
$2x = \frac{\ln 17}{\ln 3} + 4$

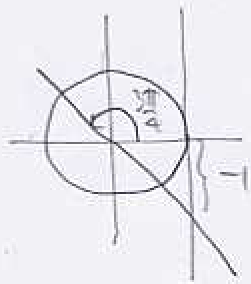
$x = \frac{1}{2}\left(\frac{\ln 17}{\ln 3} + 4\right)$

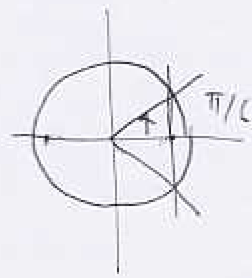
$= \frac{\ln 17}{2\ln 3} + 2 \approx 3289$

12. (6pts) Without using the calculator, find the exact values of the following expressions. Draw the unit circle and the appropriate angle under the expression.

$\sin 150° = \frac{1}{2}$ $\qquad$ $\tan \frac{3\pi}{4} = -1$ $\qquad$ $\arccos \frac{\sqrt{3}}{2} = \frac{\pi}{6}$

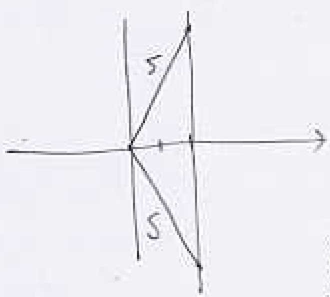






13. (5pts) If $\cos\theta = \frac{2}{5}$ and θ is in the fourth quadrant, find $\sin\theta$ and $\cos(2\theta)$.

$\frac{x}{r} = \frac{2}{5}$



$2^2 + y^2 = 5^2$

$y^2 = 21$

$y = \pm\sqrt{21}$

$y = -\sqrt{21}$

since θ is in the 4th quadrant

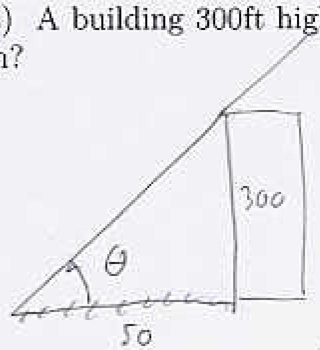
$\sin\theta = \frac{-\sqrt{21}}{5}$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \left(\frac{2}{5}\right)^2 - \left(-\frac{\sqrt{21}}{5}\right)^2$

$= \frac{4}{25} - \frac{21}{25} = -\frac{17}{25}$

14. (4pts) A building 300ft high casts a shadow 50ft long. What is the angle of elevation of the Sun?



$\tan\theta = \frac{300}{50}$

$\tan\theta = 6$

$\theta = \arctan 6 = 80.538°$

15. (5pts) Sandra is paid time-and-a-half for hours worked in excess of 40 hours. If she had weekly wages of \$442 for 48 hours worked, what is her regular hourly rate?

x= Sandra's hourly wage

$$40x + 8\left(x+\tfrac{1}{2}x\right) = 442$$

$$40x + 8\cdot\tfrac{3}{2}x = 442$$

$$40x + 12x = 442$$

$$52x = 442$$

$$x = \frac{442}{52} = 8{,}50$$

Her regular hourly rate is \$ 8,50

Bonus (7pts) After the release of radioactive material into the atmosphere from Chernobyl in 1986, the hay in parts of Europe was contaminated by iodine 131, whose half-life is 8 days. If it is all right to feed the hay to cows when 10% of the iodine 131 remains, how long do the farmers need to wait to use this hay? *(Hints: use the model $A(t) = A_0e^{kt}$ and start by finding k first.)*

Half-life = 8 days

$$\tfrac{1}{2}A_0 = A_0 e^{k\cdot 8}$$

$$\tfrac{1}{2} = e^{8k} \quad |\ln$$

$$\ln\tfrac{1}{2} = \ln e^{8k}$$

$$\ln\tfrac{1}{2} = 8k$$

$$k = \frac{\ln\frac{1}{2}}{8} \approx -0{,}0866433\ldots$$

$$0.1A_0 = A_0e^{kt}$$

$$0.1 = e^{kt} \quad |\ln$$

$$\ln 0.1 = kt$$

$$t = \frac{\ln 0.1}{k} = \frac{\ln 0.1}{\frac{\ln\frac{1}{2}}{8}} = 26{,}575 \text{ days.}$$

Need to wait 26,575 days