Fall '06/MAT 150/Exam 4 **Name:** Solution *Show all your work.*

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\sin^2\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{2}$

$\tan^2\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{1+\cos\theta}$

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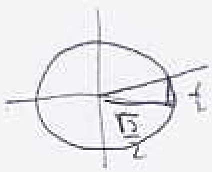
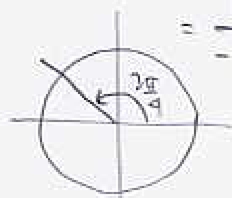
1. (8pts) Without using the calculator, find the exact values of the following expressions. Draw the unit circle and the appropriate angle under the expression.

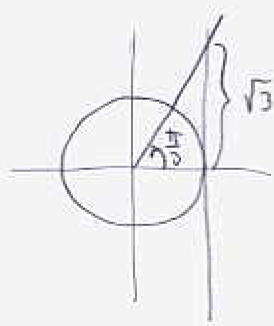
$\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\quad\sec\frac{3\pi}{4} = \frac{1}{\cos\frac{3\pi}{4}}$ $\quad\arctan\sqrt{3} = \frac{\pi}{3}$ $\quad\arcsin 1.3 =$ not defined

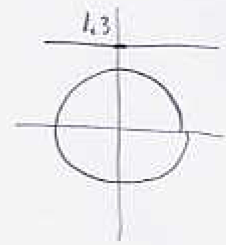




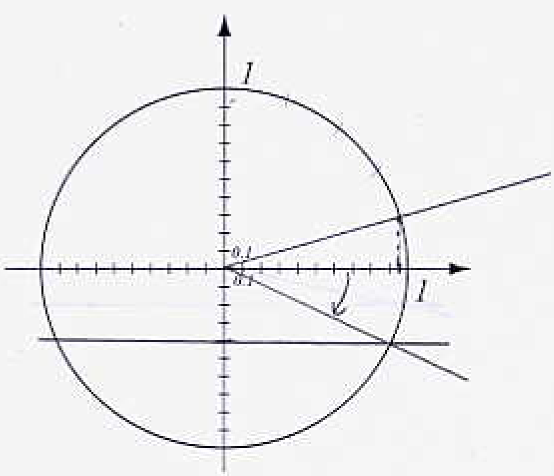
$= \frac{1}{-\frac{\sqrt{2}}{2}}$

$= -\frac{2}{\sqrt{2}} = -\sqrt{2}$





2. (4pts) Use the picture below to estimate $\cos 18^\circ$ and $\arcsin(-0.4)$ (in degrees). Then evaluate these numbers using a calculator and compare your answers.

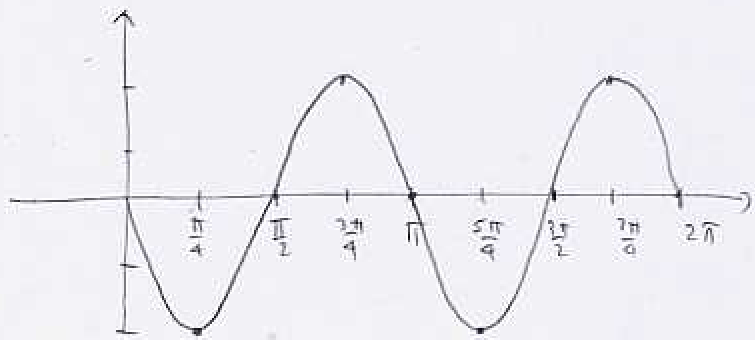


	calculator
$\cos 18^\circ \approx 0.95$	0.95
$\arcsin(-0.4) \approx -27^\circ$	-23.578°

3. (5pts) Draw two periods of the graph of $y = -3\sin(2\theta)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).

Amplitude $= |-3| = 3$

Period $= 2\pi \cdot \frac{1}{2} = \pi$



4. (5pts) Use a half-angle formula to find the exact value of $\cos 165°$.

$165° = \frac{330°}{2}$

$\cos^2 \frac{330°}{2} = \frac{1 + \cos 330°}{2}$

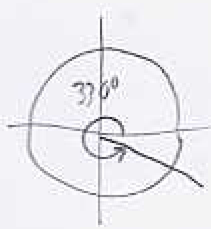
$\cos^2 165° = \frac{1 + \left(\frac{\sqrt{3}}{2}\right)}{2} \cdot \frac{2}{2}$

$\cos^2 165° = \frac{2+\sqrt{3}}{4}$

$\cos 165° = \pm\sqrt{\frac{2+\sqrt{3}}{4}}$

$\cos 165° = -\frac{\sqrt{2+\sqrt{3}}}{2}$

since $165°$ is in 2nd quadrant



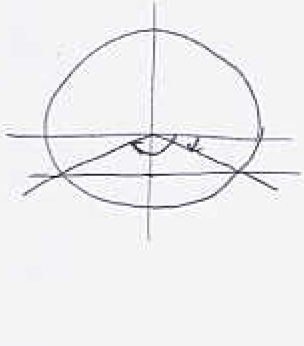
5. (4pts) Find all the solutions of the equation $2\sin\theta + 1 = 0$.

$2\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = -\frac{\pi}{6} + k \cdot 2\pi$

or $\theta = -\frac{5\pi}{6} + k \cdot 2\pi$



6. (6pts) Solve the triangle: $\alpha = 42°$, $\gamma = 57°$, $b = 5$

$\beta = 180° - (42° + 57°) = 81°$

$\frac{\sin 81°}{5} = \frac{\sin 42°}{a}$ $\frac{\sin 81°}{5} = \frac{\sin 57°}{c}$

$a \sin 81° = 5 \sin 42°$ $c = \frac{5 \sin 57°}{\sin 81°} \approx 4.246$

$a = \frac{5 \sin 42°}{\sin 81°} \approx 3.387$

7. (5pts) Show the identity: $1 - \dfrac{\sin^2\theta}{1-\cos\theta} = -\cos\theta.$

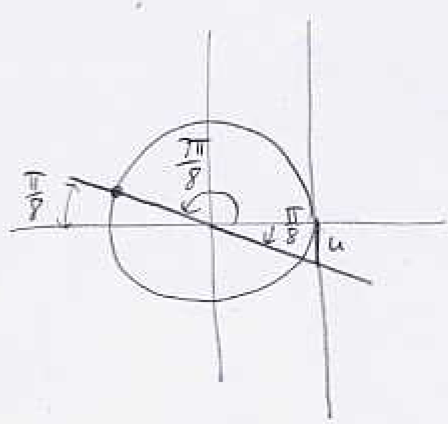
$$1 - \frac{\sin^2\theta}{1-\cos\theta} = \frac{(1-\cos\theta) - \sin^2\theta}{1-\cos\theta} = \frac{\overbrace{1-\sin^2\theta}^{\cos^2\theta} - \cos\theta}{1-\cos\theta}$$

$$= \frac{\cos^2\theta - \cos\theta}{1-\cos\theta} = \frac{-\cos\theta\cancel{(1-\cos\theta)}}{\cancel{1-\cos\theta}} = -\cos\theta$$

8. (4pts) Find the exact values of the expressions below. Draw a picture if helpful and do not use the calculator.

$\sin(\arcsin 0.57) =$ 0.57

$\arctan\left(\tan\frac{7\pi}{8}\right) = \arctan u = -\frac{\pi}{8}$

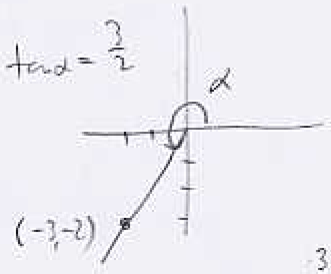


9. (9pts) Suppose that $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$ are angles so that $\tan\alpha = \frac{3}{2}$ and $\cos\beta = -\frac{3}{7}$. Use addition and double formulas to find:

a) $\sin(\alpha - \beta)$ Need: $\sin\alpha, \cos\alpha, \sin\beta, \cos\beta$

b) $\cos(2\beta)$

$\tan\alpha = \frac{3}{2}$



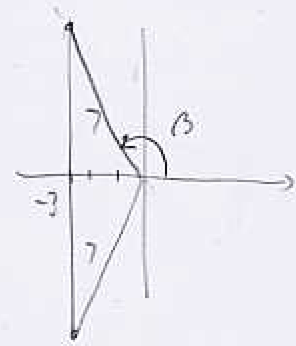
$\frac{y}{x} = \frac{3}{2} = \frac{-3}{-2}$

$r = \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{13}$

$\sin\alpha = -\frac{3}{\sqrt{13}}$ $\cos\alpha = -\frac{2}{\sqrt{13}}$

$\cos\beta = -\frac{3}{7} = \frac{x}{r}$



$(-3)^2 + y^2 = 7^2$

$9 + y^2 = 49$

$y^2 = 40$

$y = \pm\sqrt{40}$

$y = \sqrt{40} = 2\sqrt{10}$

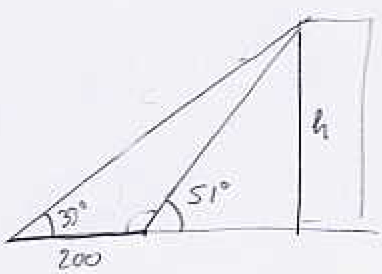
$\sin\beta = \frac{2\sqrt{10}}{7}$

a) $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

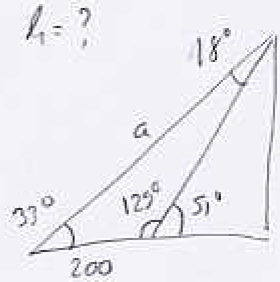
$= -\frac{3}{\sqrt{13}}\left(-\frac{3}{7}\right) - \left(-\frac{2}{\sqrt{13}}\right)\frac{2\sqrt{10}}{7} = \frac{9 + 4\sqrt{10}}{7\sqrt{13}}$

b) $\cos(2\beta) = \cos^2\beta - \sin^2\beta = \left(-\frac{3}{7}\right)^2 - \left(\frac{2\sqrt{10}}{7}\right)^2 = \frac{9 - 40}{49} = -\frac{31}{49}$

Bonus. (5pts) You take a sighting of the top of a building from a certain point and find that the angle of elevation is 33°. Then you move 200ft towards the building and take another sighting, finding the angle of elevation to be 51° now. How tall is the building?



$h = ?$



Find a first:

$\frac{\sin 129^\circ}{a} = \frac{\sin 18^\circ}{200}$

$a = \frac{200 \sin 129^\circ}{\sin 18^\circ} = 502.979 \text{ ft}$

$\frac{h}{a} = \sin 33^\circ$

$h = a \sin 33^\circ = 502.979 \cdot \sin 33^\circ = 273.942 \text{ ft}.$