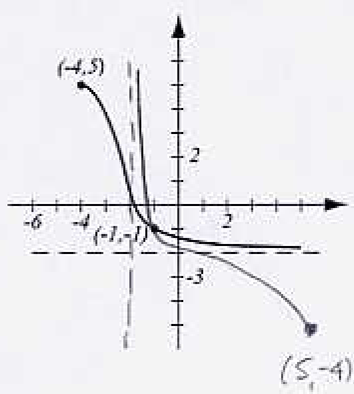
1. (3pts) Let $f(x) = x^2 + x$. Find $(f \circ f)(x)$ and simplify.

$(f \circ f)(x) = f(f(x)) = f(x^2+x) = (x^2+x)^2 + x^2+x$

$= x^4+2x^3+x^2+x^2+x$

$= x^4+2x^3+2x^2+x$

2. (4pts) The graph of f is given. Explain why f has an inverse and find the graph of its inverse function. Label the relevant points and indicate any asymptotes on the graph of f^{-1}.



f has an inverse because it is one-to-one (it passes the horizontal line test)

horizontal asymptote of $y=-2$ of f turns into a vertical asymptote $x=-2$ for f^{-1}

3. (5pts) Find the inverse of $f(x) = \dfrac{2x}{x+7}$ and the range of f.

$y = \dfrac{2x}{x+7}$

$y(x+7) = 2x$

$yx + 7y = 2x$

$7y = 2x - yx$

$7y = x(2-y)$

$x = \dfrac{7y}{2-y}$

$f^{-1}(y) = \dfrac{7y}{2-y}$

Range of f = domain of f^{-1} = $\{y \mid y \neq 2\}$

$2-y=0$

$y=2$

4. (4pts) Evaluate without using the calculator:

$\log_2 32 = 5$ $\log_4 \frac{1}{256} = -4$ $\log_{25} \frac{1}{5} = -\frac{1}{2}$ $\ln \sqrt[3]{e^7} = \frac{7}{3}$

$2^? = 32$ $4^? = \frac{1}{256}$ $25^? = \frac{1}{5}$ $e^? = \sqrt[3]{e^7} = e^{\frac{7}{3}}$

$(5^2)^? = 5^{-1}$

5. (7pts) Solve the equations

$\log_3(2x-1) = -2$

$3^{-2} = 2x-1$

$\frac{1}{9} = 2x-1$

$\frac{10}{9} = 2x$

$x = \frac{10}{2\cdot 9} = \frac{5}{9}$

$8^{x^2+x} = \left(\frac{1}{2}\right)^{3x-9}$

$(2^3)^{x^2+x} = (2^{-1})^{3x-9}$

$2^{3x^2+3x} = 2^{-3x+9}$

$3x^2+3x = -3x+9$

$3x^2+6x-9=0 \quad \div 3$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3, 1$

6. (3pts) Write as a sum of logarithms. Express powers as factors. Simplify if possible.

$\log_7 \frac{49^{x+2}}{(x-4)^5} = \log_7 49^{x+2} - \log_7 (x-4)^5$

$= (x+2)\underbrace{\log_7 49}_{=2} - 5\log_7(x-4)$

$= 2(x+2) - 5\log_7(x-4)$

7. (3pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{2}\log_3(2x-7)^4 + 3\log_3(2x-7)^3 = \log_3((2x-7)^4)^{\frac{1}{2}} + \log_3((2x-7)^3)^3$

$= \log_3(2x-7)^2 + \log_3(2x-7)^9 = \log_3((2x-7)^2(2x-7)^9)$

$= \log_3 (2x-7)^{11}$

8. (6pts) A rabbit population grows according to the law $N(t) = N_0 e^{kt}$.
a) Given that the population doubles in 18 months, find k.
b) How long does it take for the rabbit population to triple?

a) $2N_0 = N_0 e^{k \cdot 18} \quad | \div N_0$

$2 = e^{k \cdot 18} \quad | \ln$

$\ln 2 = \ln e^{k \cdot 18}$

$\ln 2 = k \cdot 18$

$k = \frac{\ln 2}{18} \approx 0.0385$

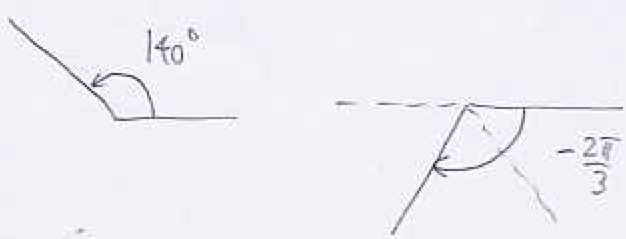
b) $3N_0 = N_0 e^{0.0385t} \quad | \div N_0$

$3 = e^{0.0385t} \quad | \ln$

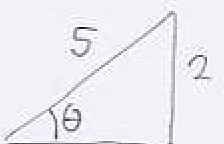
$\ln 3 = 0.0385t$

$t = \frac{\ln 3}{0.0385} = \frac{\ln 3}{\frac{\ln 2}{18}} = 28.529$ months

9. (2pts) Roughly sketch angles of measure 140° and $-\frac{2\pi}{3}$ radians.



10. (5pts) Suppose $\sin\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$. Find $\cos\theta$, $\tan\theta$ and $\sec\theta$.



$a = \sqrt{21}$

$\sin\theta = \frac{opp}{hyp} = \frac{2}{5}$

$a^2 + 2^2 = 5^2$

$a^2 + 4 = 25$

$a^2 = 21$

$a = \pm\sqrt{21}$

$\cos\theta = \frac{\sqrt{21}}{5}$

$\tan\theta = \frac{2}{\sqrt{21}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{\sqrt{21}}$

11. (3pts) Simplify using basic trigonometric identities. Do not use the calculator.

$\sin 42^\circ \csc 48^\circ - \tan 42^\circ = \sin 42^\circ \cdot \dfrac{1}{\sin 48^\circ} - \tan 42^\circ$

$$= \frac{\sin 42^\circ}{\cos 42^\circ} - \tan 42^\circ = \tan 42^\circ - \tan 42^\circ = 0$$

12. (5pts) You would like to get a wedge of pizza that is exactly 15in^2 in area. If the *diameter* of the pizza is 12in, what is the angle (in degrees) of the desired wedge?

$r = 6$ (half of diameter)

$A = \frac{1}{2} r^2 \theta$

$$15 = \frac{1}{2} 6^2 \cdot \theta$$

$$30 = 36\,\theta$$

$$\theta = \frac{30}{36} = \frac{5}{6} \text{ radians} = \frac{5}{6} \cdot \frac{180}{\pi} \text{ deg} \approx 47.746^\circ$$

Bonus (5pts) Simplify:

$\log_{\pi^2} \pi^9 = \dfrac{9}{2}$

$$(\pi^2)^x = \pi^9 \qquad 2x = 9$$

$$\pi^{2x} = \pi^9 \qquad x = \frac{9}{2}$$

$\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \dots \cdot \log_n(n+1) \cdot \log_{n+1} 2 =$ change to a single base, say e.

$$= \frac{\cancel{\ln 3}}{\ln 2} \cdot \frac{\cancel{\ln 4}}{\cancel{\ln 3}} \cdot \frac{\cancel{\ln 5}}{\cancel{\ln 4}} \cdot \dots \cdot \frac{\cancel{\ln (n+1)}}{\cancel{\ln n}} \cdot \frac{\ln 2}{\cancel{\ln (n+1)}} = \frac{\ln 2}{\ln 2} = 1$$