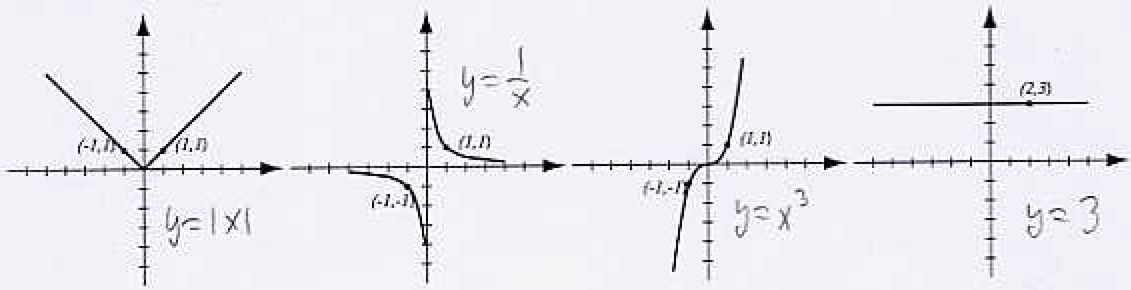
Fall '06/MAT 150/Exam 2 **Name:** Solution *Show all your work.*

1. (4pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (5pts) Let $f(x) = 3x + 7$ and $g(x) = \dfrac{5}{x^2 + 2x + 3}$. Find the following and simplify where possible:

$g(-1) = \dfrac{5}{1-2+3} = \dfrac{5}{2}$

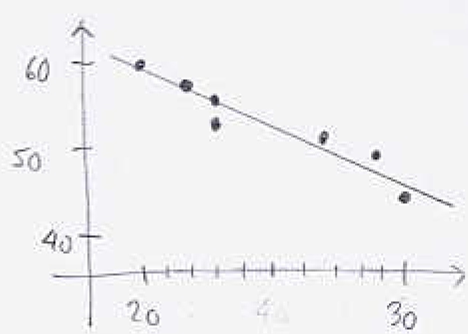
$f(2u + 4) = 3(2u+4)+7 = 6u+19$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{3x+7}{\frac{5}{x^2+2x+3}} = \dfrac{(3x+7)(x^2+2x+3)}{5}$

3. (6pts) The manager of a large clothing store wishes to find a function that relates the daily demand D for men's jeans and the price p of the jeans. The data below were obtained based on a price history of jeans sales.
a) Draw the scatterplot of the data on paper. Does the relationship look linear?
b) Use the calculator to find the "line of best fit" to the data. Draw the line on paper.
c) Interpret the slope of the line of best fit.
d) How many jeans would the store expect to sell daily if the price is \$25?

p (\$/pair)	D (pairs of jeans sold per day)
20	60
22	57
23	56
23	53
27	52
29	49
30	44

a)



Looks approximately linear

b) $y = -1.336x + 86.197$

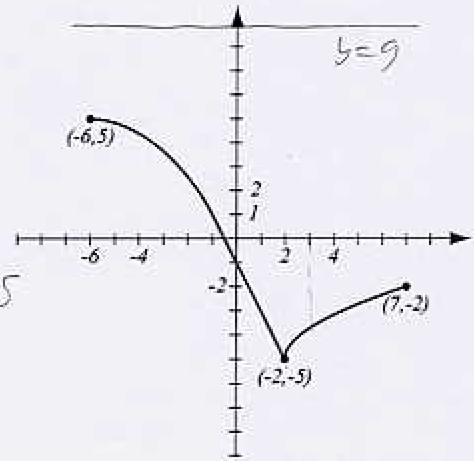
c) $m = -1.336$

For every \$1 increase in price, store sells 1.336 fewer jeans.

d) $-1.336 \cdot 25 + 86.197 = 52.797$

Approx 53 pairs of jeans

4. (6pts) Use the graph of the function f at right to answer the following questions.



a) What is $f(3)$? $f(3) = -4$

b) What are the x-intercepts? $x = -0.5$

c) Where is the function increasing? on $(-2, 7)$

d) Where does f have a local minimum? What is its value? loc. min at $x=-2$ with value $y=-5$

e) What are the solutions of the equation $f(x) = 9$? no solutions ($y=9$ doesn't intersect graph)

5. (3pts) Algebraically determine if the function $f(x) = \dfrac{x^2+2}{x^3+4x}$ even, odd or neither.

$$f(-x) = \frac{(-x)^2+2}{(-x)^3+4(-x)} = \frac{x^2+2}{-x^3-4x} = -\frac{x^2+2}{x^3+4x} = -f(x)$$

It is an odd function.

6. (7pts) The quadratic function $f(x) = -x^2 + 6x + 7$ is given. Do the following without using the calculator.

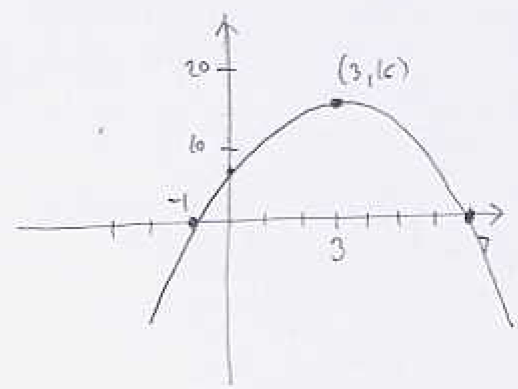
a) Find the x-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

d) What is the range of the function?

c)



a) $-x^2+6x+7=0$

$x^2-6x-7=0$

$(x-7)(x+1)=0$

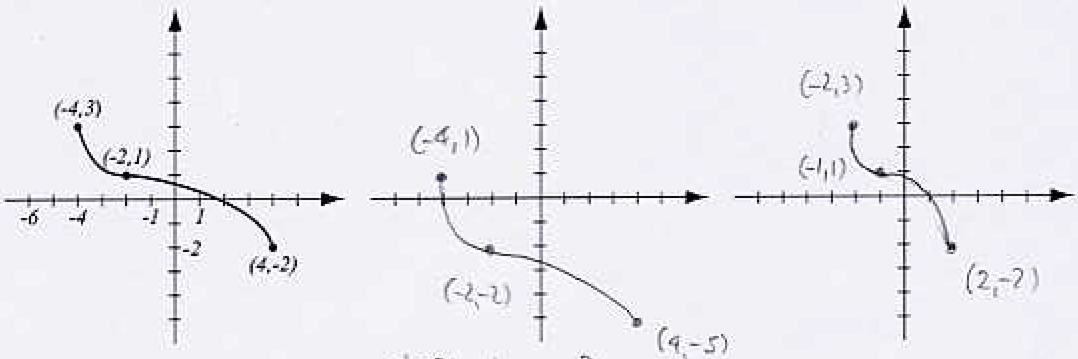
x-int: $7, -1$

d) $R = (-\infty, 16]$

b) $x = -\dfrac{6}{2\cdot(-1)} = 3$

$y = -9+18+7 = 16$

7. (4pts) The graph of $f(x)$ is drawn below. Find the graphs $f(x) - 3$ and $f(2x)$ and label all the relevant points.



shift down 3

horizontal stretch, factor $= \frac{1}{2}$

8. (8pts) Consider the rational function $f(x) = \dfrac{2x-5}{x^2-7x+10}$.

a) Find the domain of the function and the vertical asymptotes.

b) Algebraically find the x-intercepts of the graph and the y-intercept.

c) Sketch the graph of the function on paper (large and clear — make Mom proud!).

d) Find the horizontal asymptote of the graph.

a) Domain:

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

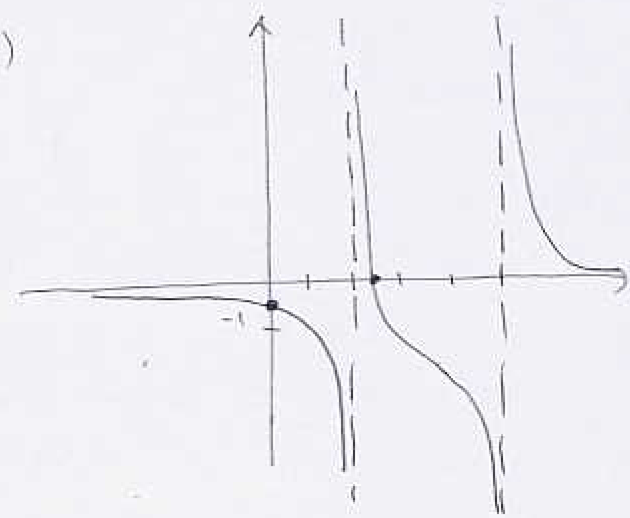
$x = 2, 5$

Domain $= \{x \mid x \neq 2 \text{ and } x \neq 5\}$

Vertical asymptotes:

$x = 2, \ x = 5$

c)



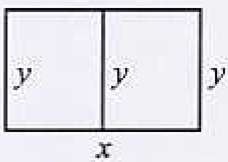
b) $2x - 5 = 0$

$x = \frac{5}{2}$ x-int

$f(0) = \dfrac{-5}{10} = -\dfrac{1}{2}$ y-int

d) $\deg P < \deg Q$, so $y = 0$ is the horizontal asymptote

9. (7pts) Farmer George has 300ft of fencing that he will use to enclose a rectangular pen and divide it in half (picture).
a) Express the area A of the pen as a function of the width x.
b) Draw a rough graph of the function $A(x)$.
c) Algebraically find the dimensions of the pen that maximize its area.



a) $2x+3y=300$

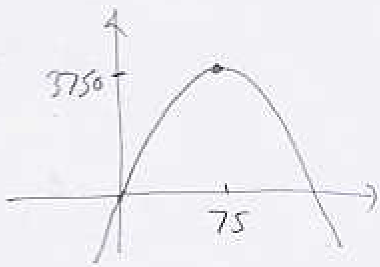
$3y = 300-2x$

$y = 100 - \frac{2}{3}x$

$A = xy = x\left(100-\frac{2}{3}x\right) = -\frac{2}{3}x^2 + 100x$

quadratic function

b)



c) need vertex of parabola

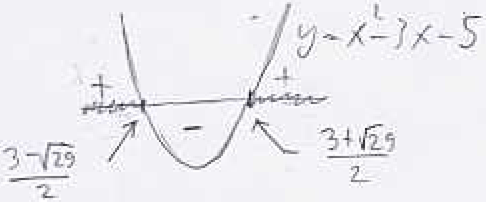
$x = -\frac{b}{2a} = -\frac{100}{-\frac{2}{3}\cdot 2} = 100\cdot\frac{3}{4} = 75$

$y = 100 - \frac{2}{3}\cdot 75 = 50$

Pen is 50×75

(max. area is 3750 sq. ft)

Bonus (5pts) Algebraically find the domain of $g(x) = \sqrt{x^2-3x-5}$. *(Hint: a graph of x^2-3x-7 will help.)*



Must have $x^2-3x-5 \geq 0$

$x^2-3x-5=0$

$x = \dfrac{3 \pm \sqrt{9-4\cdot 1\cdot(-5)}}{2}$

$= \dfrac{3 \pm \sqrt{29}}{2}$

By looking at graph, we see x^2-3x-5 is positive for shaded x's.

So domain $= \left(-\infty, \frac{3-\sqrt{29}}{2}\right] \cup \left[\frac{3+\sqrt{29}}{2}, \infty\right)$