Spring '05/MAT 250/Worksheet 2 **Name:** Solution *Show all your work.*

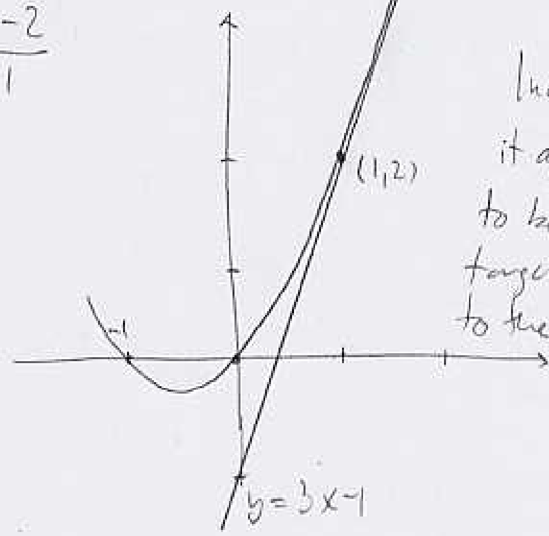
1. (6pts) The curve $y = x^2 + x$ is given.
a) Find the equation of the tangent line to the curve at point $(1, 2)$. (Use a limit to find its slope.)
b) Sketch the curve and the line you found in a) and verify that the line is indeed tangent to the curve.

a) $m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 1} \frac{x^2 + x - 2}{x - 1}$

$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{x-1}} = 1 + 2 = 3$

$y - 2 = 3(x - 1)$

$y = 3x - 1$



Indeed, it appears to be tangent to the curve

2. (9pts) It takes 5 minutes to drain, from the bottom, a cylindrical tank holding 100 liters of water. According to Torricelli's law, the volume of water remaining in the tank is $V(t) = 4(t-5)^2$, where V is in liters and t in minutes, $0 \le t \le 5$.
a) Find the instantaneous rate at which the water is draining at $t = 2$. (Use an appropriate limit). Why is the rate negative?
b) What is $V(2)$? If water were to continue draining at the same rate as you found in a), at what time t would the tank be empty?
c) Explain why it takes longer than you found in b) to actually drain the tank. A graph of V might help.

a) rate at $t=2$ $= \lim_{t \to 2} \frac{V(t) - V(2)}{t - 2} =$

$= \lim_{t \to 2} \frac{4(t-5)^2 - 36}{t - 2} =$

$= \lim_{t \to 2} \frac{4(t^2 - 10t + 25 - 9)}{t - 2}$

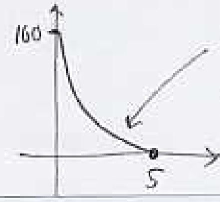
$= 4 \lim_{t \to 2} \frac{t^2 - 10t + 16}{t - 2}$

$= 4 \lim_{t \to 2} \frac{(t-8)\cancel{(t-2)}}{\cancel{t-2}} = 4(2-8) = -24\ \ell/\text{min}$ the rate slows down

b) $V(2) = 36$

If it continued to drain $24\ \ell/\text{min}$ tank would be empty in $\frac{36}{24} = 1.5$ min, so at $t = 3.5$

c) Tank actually takes 5 min to drain. Answer in b) assumed it continued to drain at the same rate, but



V decreases at a slower rate here