Fall '04/MAT 250/Exam 3 **Name:** *Show all your work.*

1. (9pts) Use L'Hospital's rule to find the limits:

a) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} =$

b) $\lim_{x\to 0}(1+x)^{-\frac{1}{x}} =$

2. (6pts) The function $f(x) = \sqrt{x}$ is given.
a) Find the linearization of this function around the point $a = 25$.
b) Use the linearization to estimate $\sqrt{26}$. How far is your estimate from $\sqrt{26}$?

3. (10pts) Let $f(x) = x^3 + 3x^2 - 24x + 2$.
a) Find the intervals of increase/decrease and where f has a local maximum and minimum.
b) Find the intervals where f is concave up or down.
c) Use your calculator and the results of a) and b) to accurately sketch the graph of f.

4. (5pts) Suppose that for a continuous and differentiable function f we have $-1 \leq f'(x) \leq 2$ for all x in $[3, 5]$ and $f(3) = 7$. Use the Mean Value Theorem to show that $5 \leq f(5) \leq 11$.

5. (6pts) Find the absolute minimum and maximum values for the function $f(x) = x^3 e^x$ on the interval $[-4, 1]$.

6. (6pts) Sketch the graph of a function defined on all reals satisfying:

$f'(x) > 0$ if $x < 0$

$f'(x) < 0$ if $x > 0$

$f'(0) = 0$, $f(0) = 4$

$f''(x) > 0$ if $x < -3$

$f''(x) < 0$ if $x > -3$

$\lim_{x \to -\infty} f(x) = 0.$

7. (8pts) Farmer Tom wants to fence in a rectangular area of 5km^2. What dimensions of the rectangle minimize the cost of the fence? Verify that your dimensions indeed give you a minimal cost.

Bonus. (5pts) Suppose f is a function that is positive ($f(x) > 0$) and concave up on $(-1, 1)$.
a) Show that $g(x) = [f(x)]^2$ is concave up on $(-1, 1)$.
b) Find a an example of a function f that is concave up, yet negative on $(-1, 1)$ for which $g = f^2$ is concave down. (Hint: think of a simple function!)