1. (9pts) Use L’Hospital’s rule to find the limits:

   a) \( \lim_{x \to \infty} \frac{\ln x}{\sqrt{x}} = \)

   b) \( \lim_{x \to 0} (1 + x)^{-\frac{1}{x}} = \)

2. (6pts) The function \( f(x) = \sqrt{x} \) is given.
   a) Find the linearization of this function around the point \( a = 25 \).
   b) Use the linearization to estimate \( \sqrt{26} \). How far is your estimate from \( \sqrt{26} \)?
3. (10pts) Let $f(x) = x^3 + 3x^2 - 24x + 2$.
   a) Find the intervals of increase/decrease and where $f$ has a local maximum and minimum.
   b) Find the intervals where $f$ is concave up or down.
   c) Use your calculator and the results of a) and b) to accurately sketch the graph of $f$.

4. (5pts) Suppose that for a continuous and differentiable function $f$ we have $-1 \leq f'(x) \leq 2$ for all $x$ in $[3, 5]$ and $f(3) = 7$. Use the Mean Value Theorem to show that $5 \leq f(5) \leq 11$. 
5. (6pts) Find the absolute minimum and maximum values for the function \( f(x) = x^3e^x \) on the interval \([-4, 1]\).

6. (6pts) Sketch the graph of a function defined on all reals satisfying:

\[ f'(x) > 0 \text{ if } x < 0 \]
\[ f'(x) < 0 \text{ if } x > 0 \]
\[ f'(0) = 0, \quad f(0) = 4 \]
\[ f''(x) > 0 \text{ if } x < -3 \]
\[ f''(x) < 0 \text{ if } x > -3 \]
\[ \lim_{x \to -\infty} f(x) = 0. \]
7. (8pts) Farmer Tom wants to fence in a rectangular area of 5km$^2$. What dimensions of the rectangle minimize the cost of the fence? Verify that your dimensions indeed give you a minimal cost.

**Bonus.** (5pts) Suppose $f$ is a function that is positive ($f(x) > 0$) and concave up on $(-1, 1)$.

a) Show that $g(x) = [f(x)]^2$ is concave up on $(-1, 1)$.

b) Find a an example of a function $f$ that is concave up, yet negative on $(-1, 1)$ for which $g = f^2$ is concave down. (Hint: think of a simple function!)