Spring '05/MAT 250/Exam 4 Name: Solution *Show all your work.*

1. (5pts) Find f if $f'(x) = e^{4x} + 5\sin x$ and $f(0) = 2$.

$$f(x) = \frac{e^{4x}}{4} - 5\cos x + C$$

$$2 = f(0) = \frac{e^0}{4} - 5\cos 0 + C$$

$$2 = \frac{1}{4} - 5 + C$$

$$7 - \frac{1}{4} = C$$

$$C = \frac{27}{4}$$

$$f(x) = \frac{e^{4x}}{4} - 5\cos x + \frac{27}{4}$$

2. (10pts) Evaluate using the Fundamental Theorem of Calculus, part 2:

a) $\int_4^8 \frac{1}{2x}\,dx = \frac{1}{2}\int_4^8 \frac{1}{x}\,dx = \frac{1}{2} \ln x \Big|_4^8 = \frac{1}{2}(\ln 8 - \ln 4)$

$$= \frac{1}{2}\ln\frac{8}{4} = \frac{\ln 2}{2}$$

b) $\int_9^{16} \sqrt{x}\,dx =$ $\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Big|_9^{16} = \frac{2}{3}x^{\frac{3}{2}}\Big|_9^{16} = \frac{2}{3}\left((\sqrt{16})^3 - (\sqrt{9})^3\right)$

$\sqrt{x} = x^{1/2}$

$$= \frac{2}{3}(64 - 27) = \frac{74}{3}$$

3. (2pts) If $\int_{-1}^{3} f(x)\,dx = 5$ and $\int_{-1}^{6} f(x)\,dx = 12$, how much is $\int_3^6 f(x)\,dx$?

$$\int_{-1}^{3} + \int_{3}^{6} = \int_{-1}^{6}$$

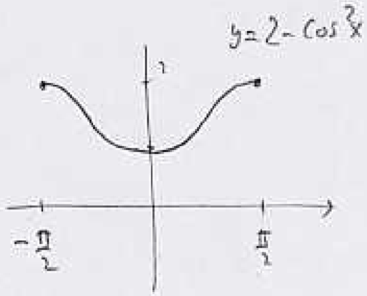
$$5 + \int_3^6 = 12$$

$$\int_3^6 f(x)\,dx = 12 - 5 = 7$$

4. (2pts) Simplify using part 1 of the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\int_1^x \sqrt[3]{t^2+t-1}\,dt = \sqrt[3]{x^2+x-1}$$

5. (4pts) Use properties of integrals to show that $\pi \le \int_{-\pi/2}^{\pi/2} 2-\cos^2 x\,dx \le 2\pi$.

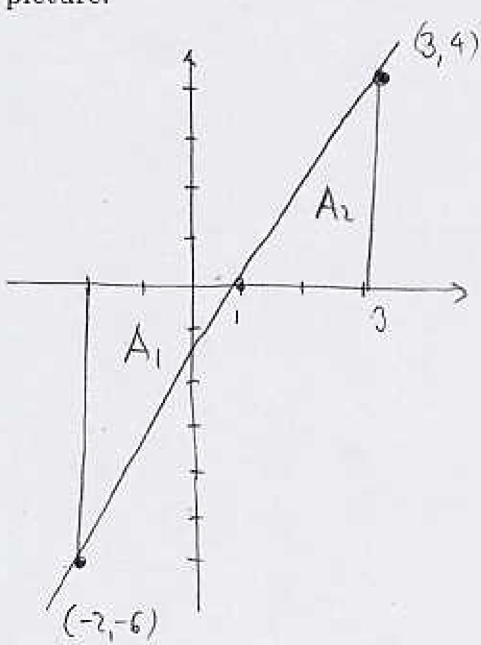


$$1 \le 2-\cos^2 x \le 2 \quad \text{for } x \text{ in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$1\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right) \le \int_{-\pi/2}^{\pi/2} 2-\cos^2 x\,dx \le 2\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right)$$

$$\pi \le \int_{-\pi/2}^{\pi/2} 2-\cos^2 x\,dx \le 2\pi$$

6. (5pts) Use the "area" interpretation of the integral to find $\int_{-2}^{3}(2x-2)\,dx$. Draw a picture.



$$\int_{-2}^{3}(2x-2)\,dx = -A_1 + A_2$$

$$= -\frac{1}{2}3\cdot 6 + \frac{1}{2}2\cdot 4$$

$$= -9 + 4 = -5$$

Spring '05/MAT 250/Exam 4, take-home portion **Name:** *Show all your work.*

The rules: you may use your book and notes on this take-home exam. Your work is to be entirely your own: you may not talk to anybody else about the exam problems. Turn the exam in on Friday, May 6th.

7. (10pts) Velocities of a vehicle were taken every 1/2 minute over a 3-minute period. The table of values is below. Assume the velocity was increasing during the whole interval.
a) Estimate the distance traveled by the vehicle by using the velocities at the beginning of each time interval.
b) Give another estimate using the velocities at the end of each time interval.
c) Draw a picture of the velocity curve. What is the geometric meaning of the quantities you computed in a) and b)?
d) Which of a) and b) is an overestimate? Underestimate?

t (min)	0	0.5	1	1.5	2	2.5	3
v (mph)	25	30	33	35	40	44	50

$\Delta t = \frac{1}{2}$ min $= \frac{1}{120}$ hr

a) estimated distance =

$= \frac{1}{120}(25+30+33+35+40+44)$

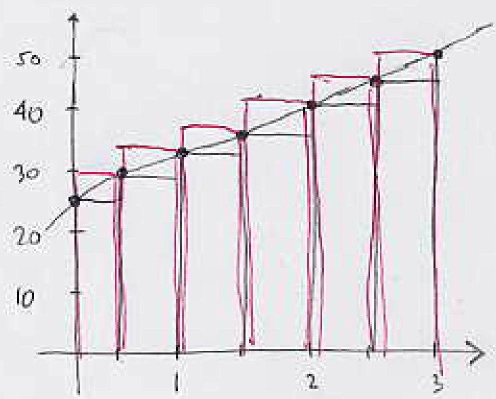
$= \frac{207}{120} \approx 1.725$ mi

b) estimated distance =

$= \frac{1}{120}(30+33+35+40+44+50)$

$= \frac{232}{120} = 1.93$ mi

c)



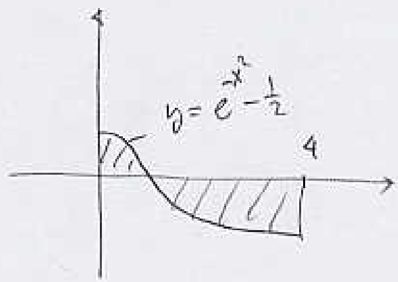
a) = area of black rectangles - underestimate

b) = area of red rectangles - overestimate

8. (4pts) Write in sigma notation.

$$\frac{3}{4}+\frac{4}{9}+\frac{5}{16}+\cdots+\frac{9}{64}+\frac{10}{81} = \sum_{i=3}^{10} \frac{i}{(i-1)^2}$$

9. (4pts) Use a graph to determine whether $\int_0^4 e^{-x^2} - \frac{1}{2}\,dx$ is positive or negative. Explain your reasoning.



$\int_0^4 (e^{-x^2} - \frac{1}{2})\,dx = A_1 - A_2 < 0$

since A_2 is clearly bigger than A_1

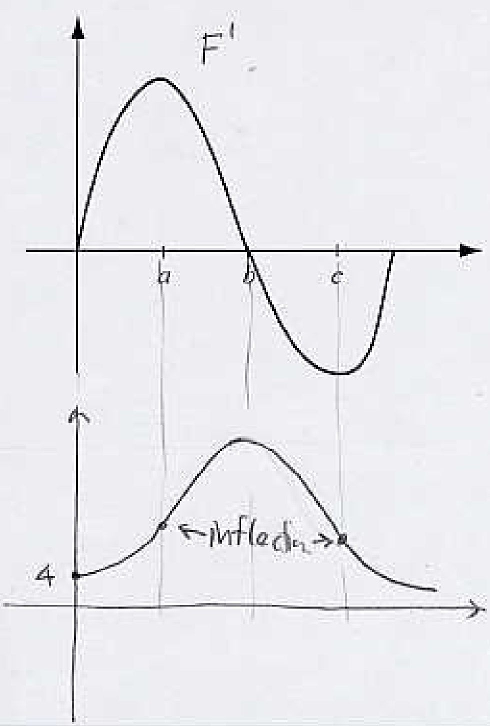
10. (4pts) Suppose the rabbit population in a certain forest is 123 rabbits at time $t = 0$ and increases at rate $r(t) = 3 + e^t$, t in years. How many rabbits are there at the end of year 4?

change in population $= \int_0^4 3 + e^t\,dt = (3t + e^t)\Big|_0^4 = 3 \cdot 4 + e^4 - 1 \approx 65.59$

Population at time $t = 4$ is $123 + 65.59 = 188.59$

approx. 189 rabbits

Bonus. (5pts) The graph of a function f is drawn below. Sketch the graph of the antiderivative F of f if we know that $F(0) = 4$.



	0		b	
F'		+	0	−
F		↗	loc. max	↘

		a		c	
F'	incr		decr.		incr.
F	conc. up	•	conc. dn	•	conc up

inflection