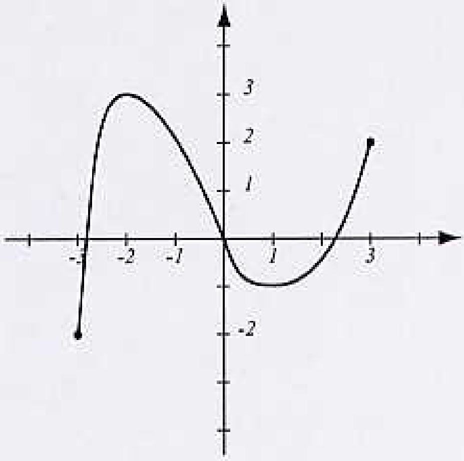
Spring '05/MAT 250/Exam 3 **Name:** *Show all your work.*

1. (6pts) The graph of the function f is given.
a) State where f has an absolute minimum and maximum value, and what the value is.
b) State where f has a local minimum and maximum value, and what the value is.



a) Abs. min at $x=-3$, $f(-3)=-2$
Abs. max at $x=-2$, $f(-2)=3$

b) Loc. min at $x=1$, $f(1)=-1$
Loc. max at $x=-2$, $f(-2)=3$

2. (9pts) Use L'Hospital's rule to find the limits:

a) $\lim\limits_{x\to0}\frac{1-\cos x}{x^2} \overset{L'H}{=} \lim\limits_{x\to0}\frac{-(-\sin x)}{2x} = \lim\limits_{x\to0}\frac{\cos x}{2} = \frac{1}{2}$

$\frac{1-1}{0}=\frac{0}{0}$ $\frac{0}{0}$

b) $\lim\limits_{x\to0}(1-2x)^{\frac{1}{x}} = e^{-2}$

$y=(1-2x)^{\frac{1}{x}}$

$\ln y = \frac{1}{x}\ln(1-2x)$

$\lim\limits_{x\to0}\frac{\ln(1-2x)}{x} \overset{L'H}{=} \lim\limits_{x\to0}\frac{\frac{1}{1-2x}(-2)}{1} = -\frac{2}{1-0} = -2$

$\frac{\ln 1}{0}=\frac{0}{0}$

3. (10pts) Let $f(x) = \ln(x^2 + 4)$.

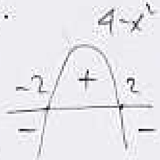
a) Find the intervals of increase/decrease and where f has a local maximum and minimum.

b) Find the intervals where f is concave up or down.

c) Use your calculator and the results of a) and b) to accurately sketch the graph of f.

a) $f'(x) = \frac{2x}{x^2+4}$

b) $f''(x) = 2 \cdot \frac{1 \cdot (x^2+4) - x \cdot 2x}{(x^2+4)^2} = \frac{2(4-x^2)}{(x^2+4)^2}$



$f'=0$: $2x=0$, $x=0$,

f' always defined

$x^2+4>0$

$f''=0$ $4-x^2=0$

$x^2=4$

$x=\pm 2$

f'' always defined

$(x^2+4)^2>0$

-2 2

f'' $-$ 0 $+$ 0 $-$

f conc dn, conc up, conc. dn

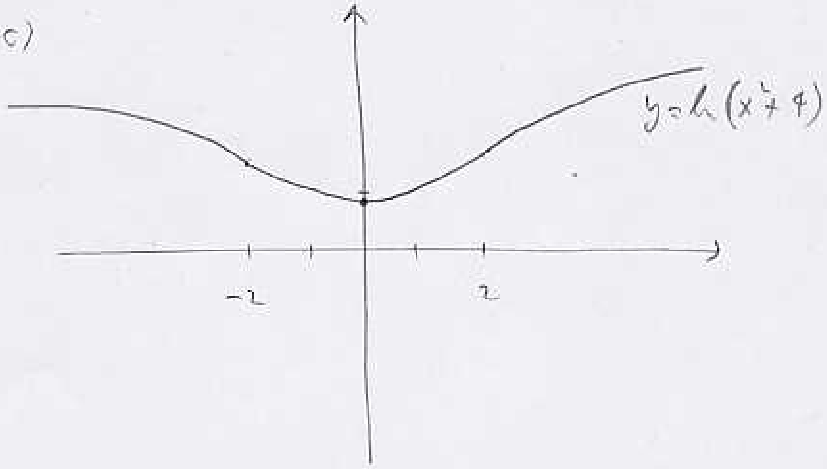
infl. pt.

0

f' $-$ 0 $+$

f ↘ loc. min ↗

c)



4. (5pts) Suppose that for a continuous and differentiable function f we have $-2 \leq f'(x) \leq 3$ for all x in $[1,4]$ and $f(1) = 7$. Use the Mean Value Theorem to show that $1 \leq f(1) \leq 16$.

By MVT, there exists a c in $(1,4)$ so that

$$f'(c) = \frac{f(4)-f(1)}{4-1}$$

Since $-2 \leq f'(c) \leq 3$ this means:

$$-2 \leq \frac{f(4)-f(1)}{3} \leq 3$$

$$-2 \leq \frac{f(4)-7}{3} \leq 3 \quad | \cdot 3$$

$$-6 \leq f(4) - 7 \leq 9 \quad | +7$$

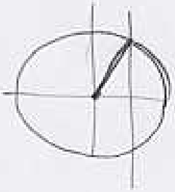
$$1 \leq f(4) \leq 16$$

5. (6pts) Find the absolute minimum and maximum values for the function $f(x) = x - 2\sin x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

$f'(x) = 1 - 2\cos x$

$1 - 2\cos x = 0$

$\cos x = \frac{1}{2}$



$x = \frac{\pi}{3}$ only sol. in $[0, \frac{\pi}{2}]$

x	$x - 2\sin x$
0	0 abs max
$\frac{\pi}{2}$	$\frac{\pi}{2} - 2 \approx -0.43$
$\frac{\pi}{3}$	$\frac{\pi}{3} - 2 \cdot \frac{\sqrt{3}}{2} = -0.68$ abs min

6. (7pts) The function $f(x) = \sqrt[4]{x}$ is given.

a) Find the linearization of this function around the point $a = 16$.

~~b) Use the linearization to estimate $\sqrt[4]{17}$.~~

b) Determine the values x for which the linear approximation is accurate to within 0.05.

a) $f'(x) = \frac{1}{4} x^{-\frac{3}{4}}$

$f'(16) = \frac{1}{4} \frac{1}{16^{3/4}} = \frac{1}{4} \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{4} \cdot \frac{1}{8} = \frac{1}{32}$

$y - 2 = \frac{1}{32}(x - 16)$

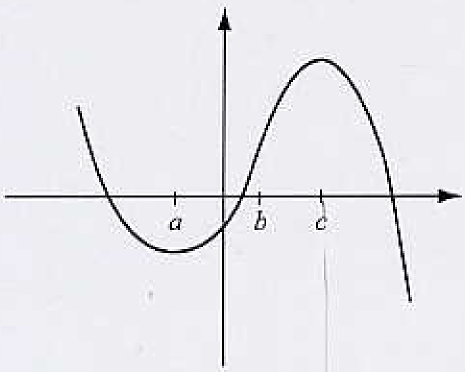
$y = \frac{1}{32}x + \frac{3}{2}$

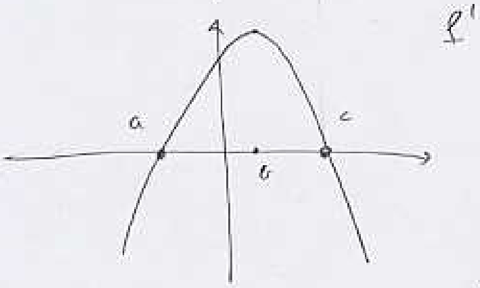
b) $-0.05 \leq \sqrt[4]{x} - \left(\frac{1}{32}x + \frac{3}{2}\right) \leq 0.05$

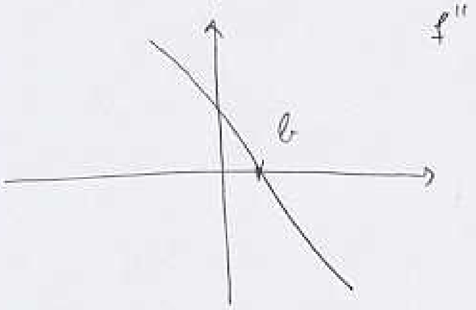
By tracing we get

$9.19 \leq x \leq 25.19$

7. ($\overset{7}{\cancel{5}}$pts) Use the graph of f to sketch the graphs of f' and f''.







Bonus. (5pts) Use Rolle's theorem to show that the equation $2x + \cos x = 0$ has at most one solution.

Suppose it has two solutions, a and b

$f(x) = 2x + \cos x$

$f(a) = f(b) = 0$ and $2x + \cos x$ is cont. and diff. everywhere, so by Rolle's thm there exists a c in (a, b) so that $f'(c) = 0$.

However: $f'(x) = 2 - \sin x$

$2 - \sin x = 0$

$\sin x = 2$ has no solution } get a contradiction, so the assumption of having two solutions is incorrect.