Spring '05/MAT 250/Exam 2 **Name:** Solution *Show all your work.*

Differentiate and simplify where appropriate:

1. (4pts) $\frac{d}{dx}(7x^3 - 4^x + \underbrace{\sqrt[3]{x^2}}_{x^{\frac{2}{3}}} + \underbrace{c^4}_{\text{constant}}) = 21x^2 - \ln 4 \cdot 4^x + \frac{2}{3}x^{-\frac{1}{3}}$

2. (4pts) $\frac{d}{dx}(x \ln x - x) = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1$

$= \ln x + 1 - 1$

$= \ln x$

3. (5pts) $\frac{d}{dx} \cos\sqrt{x^2 - 5x + 10} = -\sin\sqrt{x^2-5x+10} \cdot \left(\sqrt{x^2-5x+10}\right)'$

$= -\sin\sqrt{x^2-5x+10} \cdot \frac{1}{2\sqrt{x^2-5x+10}} \cdot (2x-5)$

$= -\frac{(2x-5)\sin\sqrt{x^2-5x+10}}{2\sqrt{x^2-5x+10}}$

4. (5pts) $\frac{d}{dx}\frac{x}{\sqrt{x^2+5}} =$

$= \frac{1 \cdot \sqrt{x^2+5} - x \cdot \frac{1}{2\sqrt{x^2+5}} \cdot 2x}{x^2+5} \cdot \frac{\sqrt{x^2+5}}{\sqrt{x^2+5}}$

$= \frac{x^2+5 - x^2}{(x^2+5)\sqrt{x^2+5}} = \frac{5}{(x^2+5)^{3/2}}$

5. (5pts) Use logarithmic differentiation to find $\frac{d}{dx} x^{\tan x}$.

$y = x^{\tan x}$

$\ln y = \tan x \cdot \ln x \quad \Big| \frac{d}{dx}$

$\frac{y'}{y} = \sec^2 x \ln x + \frac{\tan x}{x} \quad \Big| \cdot y$

$y' = x^{\tan x}\left(\sec^2 x \ln x + \frac{\tan x}{x}\right)$

6. (4pts) Find the first three derivatives of $f(x)$ and use them to find the formula for $f^{(n)}(x)$ if $f(x) = e^x(x+5)$.

$y = e^x(x+5)$

$y' = e^x(x+5) + e^x \cdot 1 = e^x(x+6)$

$y'' = e^x(x+6) + e^x \cdot 1 = e^x(x+7)$

$y''' = e^x(x+7) + e^x \cdot 1 = e^x(x+8)$

$f^{(n)}(x) = e^x(x+n+5)$

7. (6pts) Use implicit differentiation to find y'.

$e^{xy} = x^2 + y^4 \quad \Big| \frac{d}{dx}$

$e^{xy}(xy)' = 2x + 4y^3y'$

$e^{xy}(1 \cdot y + xy') = 2x + 4y^3y'$

$ye^{xy} - 2x = 4y^3y' - e^{xy}xy'$

$y' = \frac{ye^{xy} - 2x}{4y^3 - xe^{xy}}$

8. (4pts) The side x of a cube is increasing.
a) At what rate with respect to the length of the side is the volume of the cube increasing when $x = 4$cm? What are the units?
b) Approximate by how much the volume changes if x changes from 4 to 4.1 centimeters.

a) $V(x) = x^3$

$\frac{dV}{dx} = 3x^2$

$\frac{dV}{dx}(4) = 3 \cdot 4^2 = 48 \ {cm^3}/{cm}$

b) $\Delta V \approx V'(x) \cdot \Delta x$

$\Delta V \approx 48 \cdot 0.1 = 4.8 \ cm^3$

9. (6pts) An old shoe is thrown vertically upward with initial velocity 40m/s. Its position is given by $s(t) = 40t - 5t^2$, where s is in meters, t in seconds.
a) What is the maximum height that it reaches?
b) What is its velocity when, on its way down, it is at height 60m?

a) $v(t) = 40 - 10t$

Reaches max. height when $v(t) = 0$

$40 - 10t = 0$

$t = 4$

$s(4) = 40 \cdot 4 - 5 \cdot 16$

$= 80$ m

b) velocity when $s(t) = 60$

$40t - 5t^2 = 60$

$0 = 5t^2 - 40t + 60 \quad | \div 5$

$t^2 - 8t + 12 = 0$

$(t-2)(t-6) = 0$

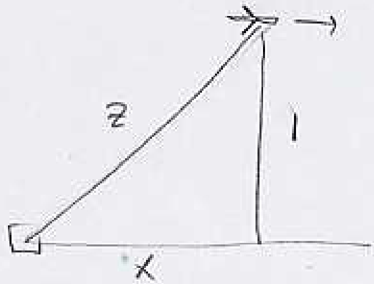
$t = 2, 6$

after $t = 4$, so on its way down

$v(6) = 40 - 10 \cdot 6$

$= -20$ m/s

10. (7pts) A plane flying horizontally at an altitude of 1mi and a speed of 500mi/hr passes directly over a radar station. Find the rate at which the straight-line distance from the plane to the radar station is increasing when this distance is 2mi.



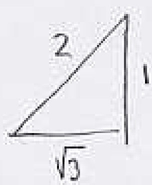
Need: $\frac{dz}{dt}$ when $z = 2$ mi

Know: $\frac{dx}{dt} = 500$ mi/hr

$$x^2 + 1 = z^2 \quad \Big| \frac{d}{dt}$$

$$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt}}{z} \qquad \left.\frac{dz}{dt}\right|_{z=2} = \frac{\sqrt{3}\cdot 500}{2} = 250\sqrt{3}$$



$x = \sqrt{3}$ when $z = 2$

Bonus. (5pts) Verify that $\underbrace{\frac{d}{dx}\left(\frac{2x^2-1}{4}\arcsin x + \frac{x\sqrt{1-x^2}}{4}\right)} = x \arcsin x$

$$\frac{d}{dx}\left(\underbrace{\frac{2x^2-1}{4}}_{\text{factor out } \frac{1}{4}\text{, differentiate}}\arcsin x + \frac{x\sqrt{1-x^2}}{4}\right) = \frac{1}{4}\left(4x\arcsin x + (2x^2-1)\frac{1}{\sqrt{1-x^2}} + 1\cdot\sqrt{1-x^2} + \frac{x\cdot(-2x)}{2\sqrt{1-x^2}}\right)$$

combine

$$= \frac{1}{4}\left(4x\arcsin x + \frac{2x^2-1-x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2}\right)$$

$$= \frac{1}{4}\left(4x\arcsin x + \underbrace{\frac{x^2-1+\overbrace{\sqrt{1-x^2}\sqrt{1-x^2}}^{1-x^2}}{\sqrt{1-x^2}}}_{=0}\right) = x\arcsin x$$