Differentiate and simplify where appropriate:

1. (4 pts) \( \frac{d}{dx} (7x^3 - 4x + \sqrt{x^2 + c^2}) = 21x^2 - \frac{\ln 4 \cdot 4x}{x^2} + \frac{2}{3} x^{-\frac{1}{3}} \)

2. (4 pts) \( \frac{d}{dx} (x \ln x - x) = x \cdot \frac{\ln x + x \cdot \frac{1}{x}}{x} - 1 \)

   \[ = \ln x + 1 - 1 \]

   \[ = \ln x \]

3. (5 pts) \( \frac{d}{dx} \cos \sqrt{x^2 - 5x + 10} = -\sin \sqrt{x^2 - 5x + 10} \cdot (\sqrt{x^2 - 5x + 10})' \)

   \[ = -\sin \sqrt{x^2 - 5x + 10} \cdot \frac{1}{2\sqrt{x^2 - 5x + 10}} \cdot (2x - 5) \]

   \[ = -\frac{(2x - 5) \sin \sqrt{x^2 - 5x + 10}}{2\sqrt{x^2 - 5x + 10}} \]

4. (5 pts) \( \frac{d}{dx} \frac{x}{\sqrt{x^2 + 5}} = \)

   \[ = \frac{1 \cdot \sqrt{x^2 + 5} - x \cdot \frac{1}{2\sqrt{x^2 + 5}} \cdot 2x}{x^2 + 5} \]

   \[ = \frac{\sqrt{x^2 + 5} - x^2}{x^2 + 5} \]

   \[ = \frac{x^2 + 5 - x^2}{(x^2 + 5)^{\frac{3}{2}}} = \frac{5}{(x^2 + 5)^{\frac{3}{2}}} \]
5. (5pts) Use logarithmic differentiation to find \( \frac{d}{dx} x^{\tan x} \).

\[
y = x^{-\sec^2 x}
\]

\[
\frac{dy}{dx} = -\sec^2 x \cdot x^{-\sec^2 x} \cdot \tan x
\]

\[
\frac{y'}{y} = \sec^2 x \cdot \tan x + \frac{\sec^2 x}{x} \cdot \frac{1}{y}
\]

6. (4pts) Find the first three derivatives of \( f(x) \) and use them to find the formula for \( f^{(n)}(x) \) if \( f(x) = e^{x(x+5)} \).

\[
y = e^{x(x+5)}
\]

\[
y' = e^{x(x+5)} + e^x \cdot 1 = e^{x(x+6)}
\]

\[
y'' = e^{x(x+6)} + e^x \cdot 1 = e^{x(x+7)}
\]

\[
y''' = e^{x(x+7)} + e^x \cdot 1 = e^{x(x+8)}
\]

\[
f^{(n)}(x) = e^{x(x+n+5)}
\]

7. (6pts) Use implicit differentiation to find \( y' \).

\[
e^{xy} = x^2 + y^2
\]

\[
\frac{d}{dx} e^{xy} = \frac{d}{dx} (x^2 + y^2)
\]

\[
e^{xy} \cdot (xy)' = 2x + 4y^3 y'
\]

\[
e^{xy} \cdot (y + xy') = 2x + 4y^3 y'
\]

\[
y e^{xy} - 2x = 4y^3 y' - e^{xy} y'
\]

\[
y' = \frac{y e^{xy} - 2x}{4y^3 - xe^{xy}}
\]
8. (4pts) The side $x$ of a cube is increasing.
   a) At what rate with respect to the length of the side is the volume of the cube increasing when $x = 4\text{ cm}$? What are the units?
   b) Approximate by how much the volume changes if $x$ changes from 4 to 4.1 centimeters.

   \[
   a) \quad V(x) = x^3 \\
   \frac{dV}{dx} = 3x^2 \\
   \frac{dV}{dx}(4) = 3 \cdot 4^2 = 48 \text{ cm}^3/\text{cm}
   \]

   \[
   b) \quad \Delta V \approx V(x) \cdot \Delta x \\
   \Delta V \approx 48 \cdot 0.1 = 4.8 \text{ cm}^3
   \]

9. (6pts) An old shoe is thrown vertically upward with initial velocity 40m/s. Its position is given by $s(t) = 40t - 5t^2$, where $s$ is in meters, $t$ in seconds.
   a) What is the maximum height that it reaches?
   b) What is its velocity when, on its way down, it is at height 60m?

   \[
   a) \quad v(t) = 40 - 10t \\
   \text{Reaches max. height when } v(t) = 0 \\
   40 - 10t = 0 \\
   t = 4 \\
   s(4) = 40 \cdot 4 - 5 \cdot 16 \\
   = 80 \text{ m}
   \]

   \[
   \text{b) velocity when } s(t) = 60 \\
   40t - 5t^2 = 60 \\
   0 = 5t^2 - 40t + 60 \\
   t = 2, 6 \\
   v(6) = 40 - 10 \cdot 6 \\
   = -20 \text{ m/s}
   \]
10. (7pts) A plane flying horizontally at an altitude of 1mi and a speed of 500mi/hr passes directly over a radar station. Find the rate at which the straight-line distance from the plane to the radar station is increasing when this distance is 2mi.

\[ \text{Need: } \frac{dz}{dt} \text{ when } z = 2 \text{mi} \]

\[ \text{Know: } \frac{dx}{dt} = 500 \text{ mi/hr} \]

\[ x^2 + 1 = z^2 \]

\[ \frac{d}{dt} \]

\[ 2x \frac{dx}{dt} = 2z \frac{dz}{dt} \]

\[ \frac{dz}{dt} = \frac{x \frac{dx}{dt}}{z} \]

\[ \left. \frac{dz}{dt} \right|_{z=2} = \frac{\sqrt{3} \cdot 500}{2} = 250\sqrt{3} \]

\[ x = \sqrt{3} \]

\[ \text{when } z = 2 \]

**Bonus. (5pts) Verify that**

\[ \frac{d}{dx} \left( \frac{2x^2 - 1}{4} \arcsin x + \frac{x\sqrt{1-x^2}}{4} \right) = x \arcsin x \]

\[ \frac{d}{dx} \left( \frac{2x^2 - 1}{4} \arcsin x + \frac{x\sqrt{1-x^2}}{4} \right) = \frac{1}{4} \left( 4x \arcsin x + (2x^2-1) \frac{1}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{2} \right) \]

\[ \text{factor out } \frac{1}{4} \]

\[ \text{diff. inside} \]

\[ = \frac{1}{4} \left( 4x \arcsin x + \frac{2x^2 - 1 - x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2} \right) \]

\[ = \frac{1}{4} \left( 4x \arcsin x + \frac{x^2 - 1 + \sqrt{1-x^2} \sqrt{1-x^2}}{\sqrt{1-x^2}} \right) = x \arcsin x \]

\[ = 0 \]