Spring '05/MAT 250/Exam 1 **Name:** Solution *Show all your work.*

1. (7pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

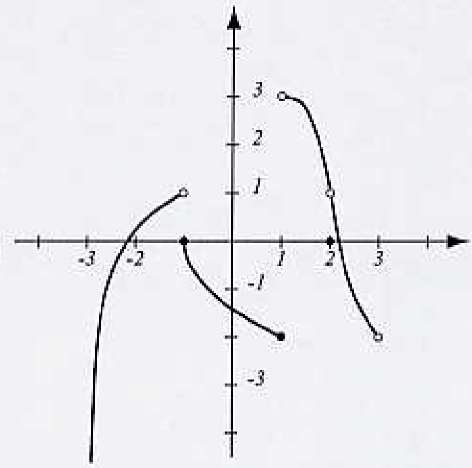
$\lim_{x \to -3^+} f(x) = -\infty$

$\lim_{x \to 3^-} f(x) = -2$

$\lim_{x \to -1} f(x) = $ d.n.e. (one-sided limits not equal)

$\lim_{x \to 1+} f(x) = 3$

$\lim_{x \to 2} f(x) = 1$



Why is f not continuous at $x = 2$?

Because $\underbrace{\lim_{x\to 2} f(x)}_{1} \neq f(1)$

2. (7pts) Consider the equation $e^x - x^2 = 0$.

a) Use the Intermediate Value Theorem to show that this equation has a solution in the interval $[-2, 0]$.

b) Use your calculator to find an interval of length 0.01 that contains this root. Explain why the IVT will guarantee there is root in the interval that you found.

a) Let $f(x) = e^x - x^2$

f is continuous on $\mathbb{R}$.

$f(-2) = e^{-2} - 4 = -3.86$

$f(0) = 1 - 0 = 1$

Since $-3.86 < 0 < 1$, by IVT there is a point c in $(-2, 0)$ so that $f(c) = 0$.

b) By tracing and zooming we find:

	x	$f(x)$
0.01 apart	-0.70	0.007
	-0.71	-0.12

Since $-0.12 < 0 < 0.007$, by IVT there is a point c in $(-0.71, -0.70)$ so that $f(c) = 0$

3. (13pts) Find the following limits algebraically.

a) $\lim_{x\to 4} \frac{x^2+3x-28}{x-4} = \lim_{x\to 4} \frac{(x+7)\cancel{(x-4)}}{\cancel{x-4}} = 4+7=11$

b) $\lim_{x\to\infty} \frac{x^2-3x+1}{x^3+3x^2-x} = \lim_{x\to\infty} \frac{x^2\left(1-\frac{3}{x}+\frac{1}{x^2}\right)}{x^3\left(1+\frac{3}{x}-\frac{1}{x^2}\right)} = \lim_{x\to\infty} \frac{1}{x}\,\frac{\left(1-\frac{3}{x}+\frac{1}{x^2}\right)}{\left(1+\frac{3}{x}-\frac{1}{x^2}\right)}$

$= \frac{1}{\infty}\cdot\frac{(1-0+0)}{1+0-0} = 0\cdot 1 = 0$

c) $\lim_{x\to 4^-} \frac{1}{x^2-16} = \frac{1}{0^-} = -\infty$

$\uparrow$

when $x<4$, $x^2-16<0$

4. (5pts) Find $\lim_{x\to 0} x^8 \cos\left(2005+\frac{1}{x}\right)$. Use the theorem that rhymes with what water does, when it is cold.

$-1 \le \cos\left(2005+\frac{1}{x}\right) \le 1 \quad |\cdot x^8$

$-x^8 \le x^8\cos\left(2005+\frac{1}{x}\right) \le x^8$

$\left.\begin{array}{l} \lim_{x\to 0}(-x^8)=0 \\ \lim_{x\to 0} x^8 = 0 \end{array}\right\}$ By the squeeze theorem,

$\lim_{x\to 0} x^8\cos\left(2005+\frac{1}{x}\right) = 0$

5. (7pts) The curve $y = \sqrt{x+1}$ is given.
a) Find the equation of the tangent line to the curve at point $(3, 2)$.
b) Sketch the curve and the line you found in a) and verify that the line is indeed tangent to the curve.

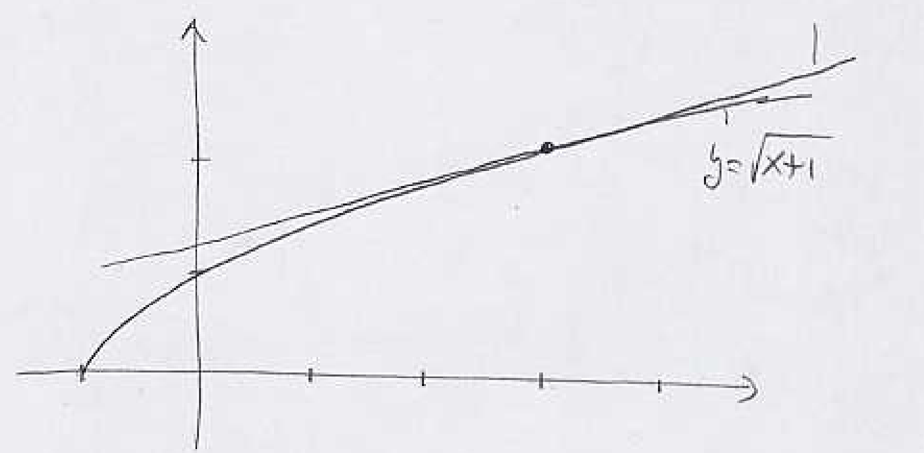
a) $$m = \lim_{x\to 3} \frac{\sqrt{x+1} - \sqrt{3+1}}{x-3} = \lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to 2} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)} = \frac{1}{\sqrt{3+1}+2} = \frac{1}{4}$$

$y = \frac{1}{4}x + \frac{5}{4}$

$$y - 2 = \frac{1}{4}(x-3)$$

$$y = \frac{1}{4}x + \frac{5}{4}$$



6. (5pts) Sketch the graph of a function defined on $(-2, 4)$ and continuous on that whole interval with the following properties:

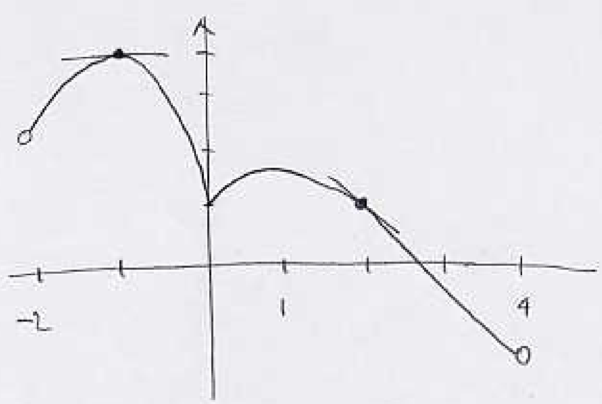
$f(-1) = 4$

$f'(-1) = 0$

$f(2) = 1$

$f'(2) = -1$

$f'(0)$ d.n.e.
↑
sharp point



7. (6pts) According to experiments, a falling object is approximately $f(t) = 5t^2$ meters below the point of release after t seconds of travel.

a) Find the average velocity for the time period beginning with $t = 2$ and lasting:

0.1s | 0.01s | 0.001s

$\bar{v}_{0.1} = \dfrac{5 \cdot 2.1^2 - 5 \cdot 2^2}{0.1}$

$= 20.5$

$\bar{v}_{0.01} = \dfrac{5 \cdot 2.01^2 - 20}{0.01}$

$= 20.05$

$\bar{v}_{0.001} = \dfrac{5 \cdot 2.001^2 - 20}{0.001}$

$= 20.005$

b) What is the instantaneous velocity at time $t = 2$?

$v(2) = 20 \text{ m/s}$

Bonus. (5pts) Show that $\infty - \infty$ is an indeterminate form. For that purpose, give two examples of functions f and g, which satisfy $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to\infty} g(x) = \infty$ in both cases, but in one case $\lim_{x\to\infty}(f(x) - g(x)) = 2$, and in the other case $\lim_{x\to\infty}(f(x) - g(x)) = \infty$. (Hint: think simple.)

a) $f(x) = x + 2$

$g(x) = x$

$\lim_{x\to\infty}(f(x) - g(x)) = \lim_{x\to\infty}(x+2) - x = 2$

e) $f(x) = x^2$

$g(x) = x$

$\lim_{x\to\infty}(f(x) - g(x)) = \lim_{x\to\infty} x^2 - x$

$= \lim_{x\to\infty} x^2\left(1 - \frac{1}{x}\right)$

$= \infty \cdot (1 - 0) = \infty$