

1. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_2(x^4 \sqrt{x+1}) = \log_2 x^4 + \log_{(x+1)^{1/2}} \sqrt{x+1} = 4\log_2 x + \frac{1}{2} \log_2(x+1)$$

$$\ln \frac{e^x}{(e^x + 1)^2} = \ln e^x - \ln(e^x + 1)^2 = x - 2\ln(e^x + 1)$$

2. (6pts) Write each the following as a single logarithm. Simplify if possible.

$$\begin{aligned} \frac{1}{2} \log_6 v^4 - 2 \log_6 v^2 &= \log_6 (v^4)^{1/2} - \log_6 (v^2)^2 = \log_6 v^2 - \log_6 v^4 = \log_6 \frac{v^2}{v^4} \\ &= \log_6 v^{-2} = -2 \log_6 v \end{aligned}$$

$$\begin{aligned} \ln(x^2 - 1) - 2 \ln(x + 1) &= \ln(x^2 - 1) - \ln(x + 1)^2 = \ln \frac{x^2 - 1}{(x + 1)^2} = \ln \frac{(x - 1)(x + 1)}{(x + 1)^2} \\ &= \ln \frac{x - 1}{x + 1} \end{aligned}$$

3. (2pts) Compute the following number using your calculator. Show how you obtained your number.

$$\log_3 15 = \frac{\log 15}{\log 3} \approx 2.46$$

4. (10pts) Solve the equations:

$$2^{3x-1} = 5^{x+1} \quad | \ln$$

$$\ln 2^{3x-1} = \ln 5^{x+1}$$

$$(3x-1)\ln 2 = (x+1)\ln 5$$

$$3x\ln 2 - \ln 2 = x\ln 5 + \ln 5$$

$$3x\ln 2 - x\ln 5 = \ln 2 + \ln 5$$

$$x(3\ln 2 - \ln 5) = \ln 2 + \ln 5$$

$$x = \frac{\ln 2 + \ln 5}{3\ln 2 - \ln 5} \approx 4.12$$

$$\log_4 x + \log_4(x-3) = 1$$

$$\log_4 x(x-3) = 1$$

$$x(x-3) = 4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4 \text{ or } x = -1$$

can't be
plugged in $\log_4 x$
so not a solution.

5. (7pts) A culture of bacteria obeys the law of uninhibited growth. Suppose 500 bacteria were present initially and there are 1200 after 7 hours.

a) Write the function that describes the number of bacteria after t hours. What is the growth rate?

b) When will the bacteria count reach 15,000?

$$a) A = A_0 e^{kt}$$

$$A_0 = 500$$

$$1200 = 500 e^{k \cdot 7} \quad | \div 500$$

$$2.4 = e^{7k} \quad | \ln$$

$$\ln 2.4 = 7k$$

$$k = \frac{\ln 2.4}{7} \approx 0.125066 \dots$$

$$\approx 0.1251$$

$$A(t) = 500 e^{0.1251t}$$

$$\text{growth rate} = 12.51\%$$

$$b) 15000 = 500 e^{0.1251t} \quad | \div 500$$

$$30 = e^{0.1251t} \quad | \ln$$

$$\ln 30 = 0.1251t$$

$$t = \frac{\ln 30}{0.1251} = 27.2 \text{ hours}$$