Fall '05/MAT 150/Worksheet 7 **Name:** Solution *Show all your work.*

1. (5pts) Solve the equation: $9^{x^2-3x} = \left(\frac{1}{3}\right)^{x+2}$.

$(3^2)^{x^2-3x} = (3^{-1})^{x+2}$

$3^{2x^2-6x} = 3^{-x-2}$

$2x^2-6x = -x-2$

$2x^2-5x+2=0$

$x = \frac{5 \pm \sqrt{25-4\cdot2\cdot2}}{2\cdot2} = \frac{5\pm\sqrt{9}}{4} = 2, \frac{1}{2}$

2. (4pts) Evaluate without using the calculator:

$\log_2 32 = 5$ $2^? = 32$

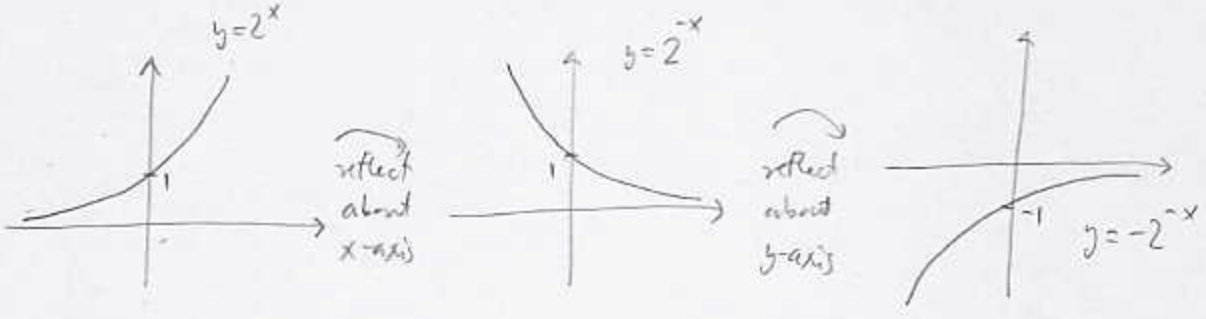
$\log_5 \frac{1}{125} = -3$ $5^? = \frac{1}{125}$

$\log_7 \sqrt[5]{7^6} = \frac{6}{5}$ $7^? = 7^{\frac{6}{5}}$

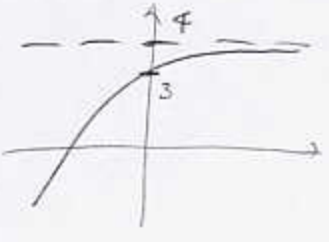
$\log_a \sqrt{a} = \frac{1}{2}$ $a^? = a^{\frac{1}{2}}$

3. (7pts) The function $f(x) = 4 - 2^{-x}$ is given.
a) Explain how the graph of $y = 2^x$ should be transformed in order to get the graph of f. Then perform the transformations on paper to obtain the graph of f.
b) What are the domain and range of f? What is the horizontal asymptote of the graph?

a)



shift up 4 →



b) Domain = $\mathbb{R}$

Range = $(-\infty, 4)$

Horizontal asymptote: $y = 4$

4. (8pts) Solve the equations:

$\log_x 4 = 5$

$x^5 = 4$

$x = \sqrt[5]{4}$

$\ln x = 4$

$e^4 = x$

$10^{2x+3} = 56$

$2x+3 = \log 56$

$2x = \log 56 - 3$

$x = \frac{\log 56 - 3}{2} \approx -0.6259$

$\log_3(2x-17) = 4$

$2x - 17 = 3^4$

$2x = 98$

$x = 49$

5. (6pts) Between 12:00PM and 1:00PM cars arrive at Murray Bank's drive-thru at the rate of twelve cars per hour (0.2 cars per minute). The formula $P(t) = 1 - e^{-0.2t}$ can be used to determine the probability that a car will arrive within t minutes of 12:00PM.
a) What is the probability that a car will arrive in the first 25 minutes?
b) How many minutes are needed for the probability to reach 50%?

a) $P(25) = 1 - e^{-0.2 \cdot 25} = 1 - e^{-5} = 0.9932$

Probability is 99.32 %

b) $0.5 = 1 - e^{-0.2t} \quad | -1$

$-0.5 = -e^{-0.2t} \quad | \cdot (-1)$

$0.5 = e^{-0.2t} \quad | \ln$

$\ln 0.5 = -0.2t$

$t = \frac{\ln 0.5}{-0.2} = 3.47$ min.