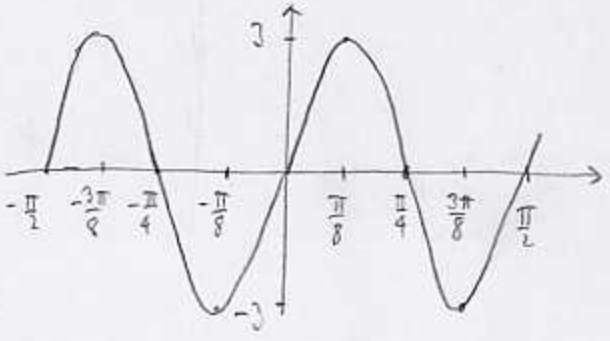
Fall '05/MAT 150/Worksheet 10 **Name:** Solution *Show all your work.*

1. (4pts) Draw two periods of the graph of $y = 3\sin(4x)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).

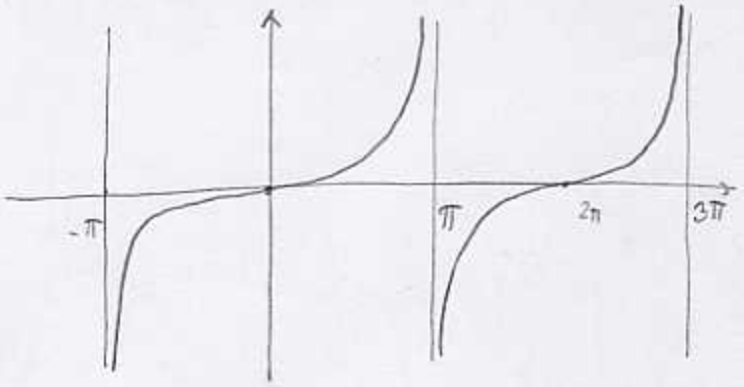
amplitude = 3

period = $2\pi \cdot \frac{1}{4} = \frac{\pi}{2}$



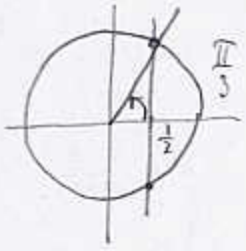
2. (4pts) Draw two periods of the graph of $y = \tan\left(\frac{1}{2}x\right)$. What is the period? Indicate where the special points are (x-intercepts, asymptotes).

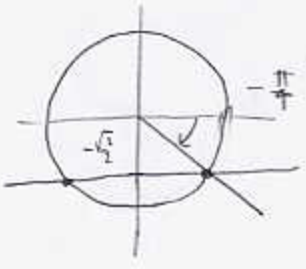
period = $\pi \cdot \frac{1}{\frac{1}{2}} = 2\pi$

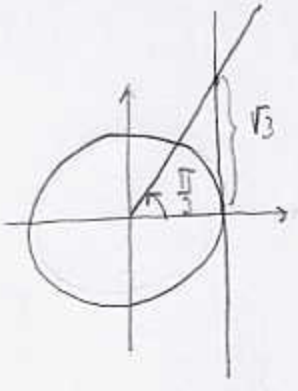


3. (6pts) Without using the calculator, find the exact values of the following inverse trigonometric functions. Draw the unit circle and the appropriate angle under the expression.

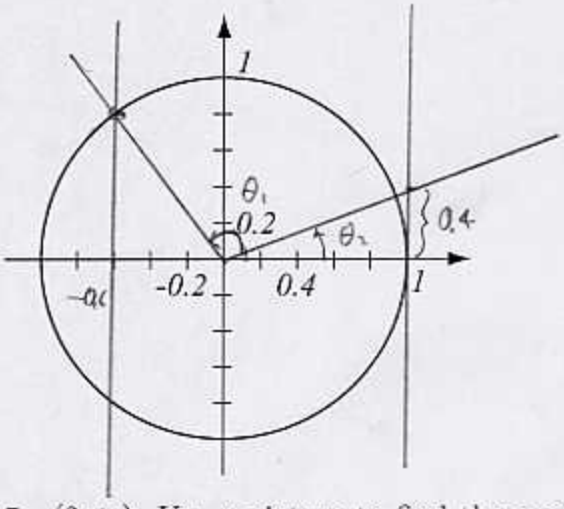
$\arccos\frac{1}{2} = \frac{\pi}{3}$ $\quad\quad$ $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ $\quad\quad$ $\arctan\sqrt{3} = \frac{\pi}{3}$







4. (4pts) Use the picture below to estimate (in degrees) $\arccos(-0.6)$ and $\arctan(0.4)$. Then evaluate these numbers using a calculator and compare your answers.



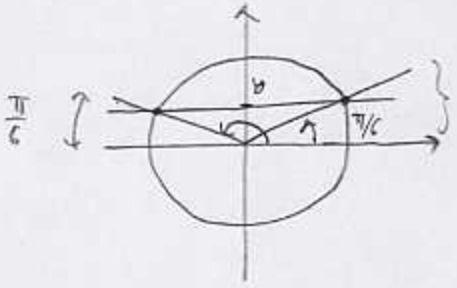
Calculator:

$\arccos(-0.6) \approx 130^\circ$ $\quad$ 126.87°

$\arctan 0.4 \approx 20^\circ$ $\quad$ 21.80°

5. (3pts) Use a picture to find the exact value below. Do not use the calculator.

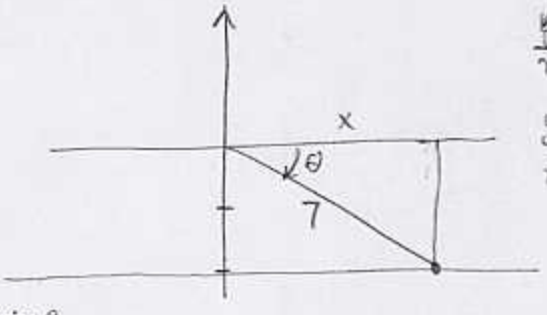
$$\arcsin\left(\underbrace{\sin\left(\frac{5\pi}{6}\right)}_{=y}\right) = \arcsin y = \frac{\pi}{6}$$



This angle have the same sine as $\frac{5\pi}{6}$ and lies in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

6. (4pts) Find the exact value of the expression below.. Draw a picture and do not use the calculator.

$$\tan\left(\arcsin\left(-\frac{2}{7}\right)\right) = \tan\theta = \frac{y}{x} = -\frac{2}{3\sqrt{5}}$$



$\frac{y}{r} = -\frac{2}{7}$

$y = -2$ $\quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$r = 7$

$x^2 + (-2)^2 = 7^2$

$x^2 = 45$

$x = \sqrt{45} = 3\sqrt{5}$

7. (5pts) Show the identity: $\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta} = 2\sec\theta.$

$$\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = \frac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)\cos\theta} =$$

$$= \frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{(1+\sin\theta)\cos\theta} = \frac{1+1+2\sin\theta}{(1+\sin\theta)\cos\theta} = \frac{2(1+\sin\theta)}{(1+\sin\theta)\cos\theta} = \frac{2}{\cos\theta} = 2\sec\theta$$