Final Exam

Fall '05/MAT 150/~~Exam 4~~ **Name:** Solution *Show all your work.*

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

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1. (4pts) Find the equation of the line whose x-intercept is 2 and that is parallel to the line $3x + y = 4$. Draw both lines in coordinate system.

$3x+y=4$

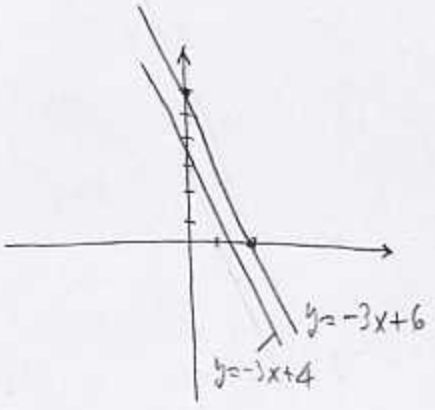
$y=-3x+4$

slope $=-3$

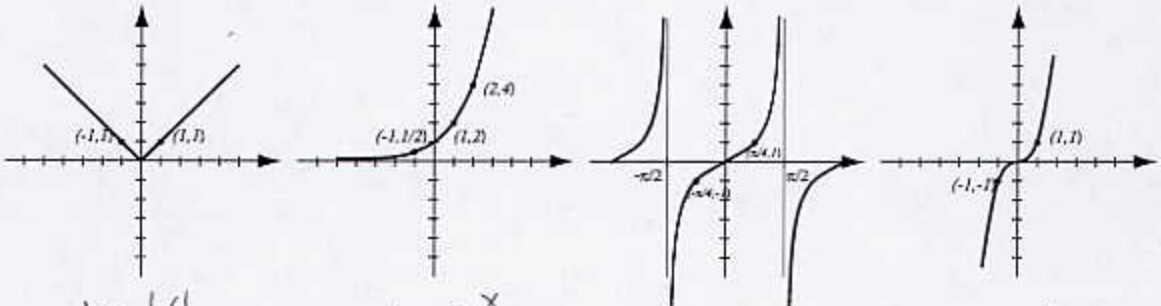
Other line contains $(2,0)$

$y-0=-3(x-2)$

$y=-3x+6$



2. (4pts) The following are graphs of basic functions that we have had in this course. Write the equation of the graph under each one.



$y=|x|$ $y=2^x$ $y=\tan x$ $y=x^3$

3. (4pts) Solve the equation $x^2-4x+8=0$. Write down the complex solutions if applicable.

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} = \frac{4 \pm \sqrt{16-32}}{2} = \frac{4 \pm \sqrt{-16}}{2} = \frac{4 \pm 4i}{2}$$

$$= 2 \pm 2i$$

4. (7pts) The quadratic function $f(x) = x^2 + 3x - 10$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

d) What is the range of the function?

a) $x^2 + 3x - 10 = 0$

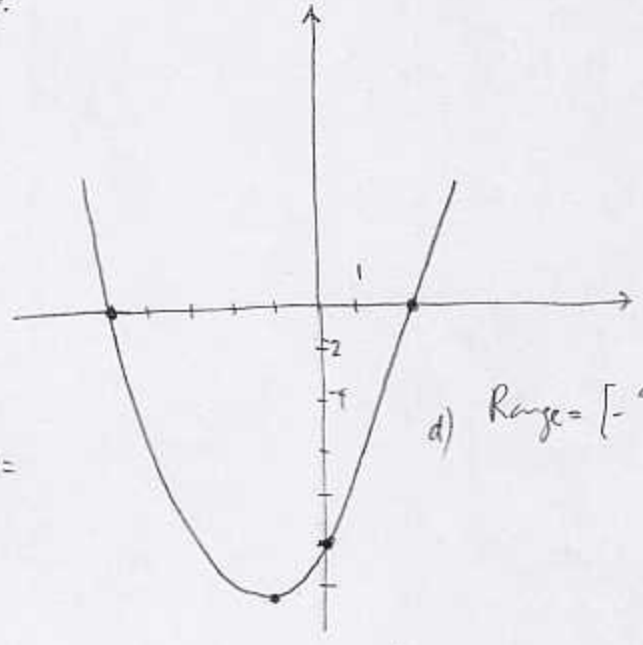
$(x+5)(x-2) = 0$

$x = -5, 2$

b) $x = -\frac{3}{2 \cdot 1} = -\frac{3}{2}$

$y = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) - 10 = \frac{9}{4} - \frac{9}{2} - 10 =$

$= \frac{9 - 18 - 40}{4} = -\frac{49}{4} = -12\frac{1}{4}$



d) Range $= \left[-\frac{49}{4}, \infty\right)$

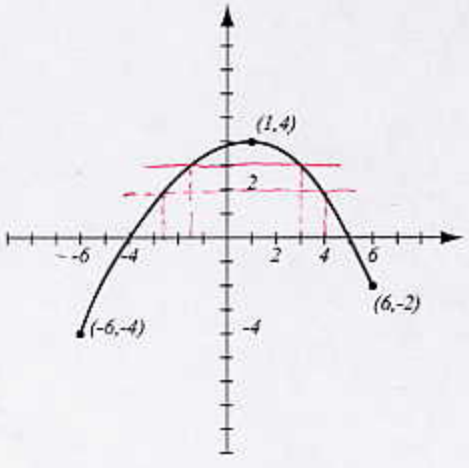
5. (7pts) Use the graph of the function f, at right, to answer the following questions.

a) What are the x-intercepts? $x = -4, 5$

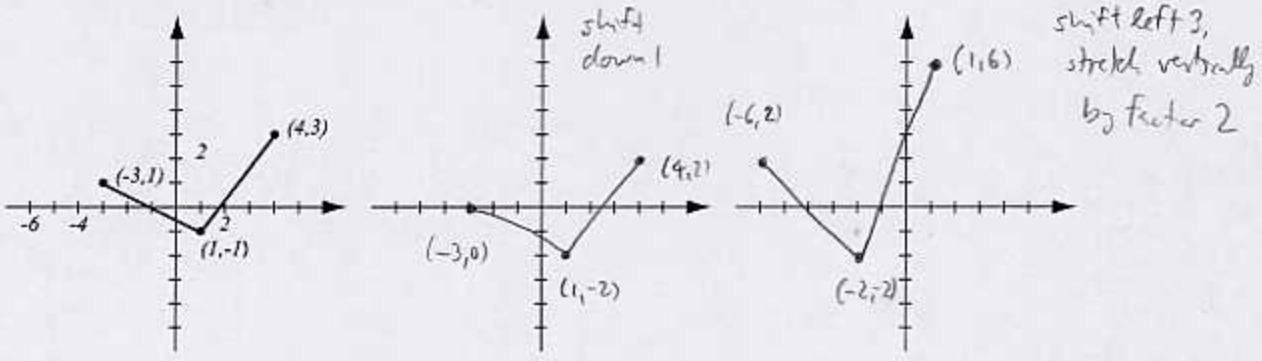
b) Where is the function decreasing? on $(1, 6)$

c) Where does f have a local maximum? What is its value? at $x=1$, value $=4$

d) What are the solutions of the equation $f(x) = \frac{3}{2}$? $x = 4, -2.7$



6. (5pts) The graph of $f(x)$ is drawn below. Find the graphs $f(x) - 1$ and $2f(x+3)$ and label all the relevant points.



7. (4pts) Evaluate without using the calculator:

$\log_5 25 = 2$ $\quad$ $\log_3 \frac{1}{9} = -2$ $\quad$ $\log \sqrt{1000} = \frac{3}{2}$ $\quad$ $\ln \sqrt[7]{e^5} = \frac{5}{7}$

$5^? = 25$ $\quad$ $3^? = \frac{1}{9}$ $\quad$ $10^? = \sqrt{10^3} = 10^{\frac{3}{2}}$ $\quad$ $e^? = e^{\frac{5}{7}}$

8. (4pts) Write as a single logarithm. Simplify if possible.

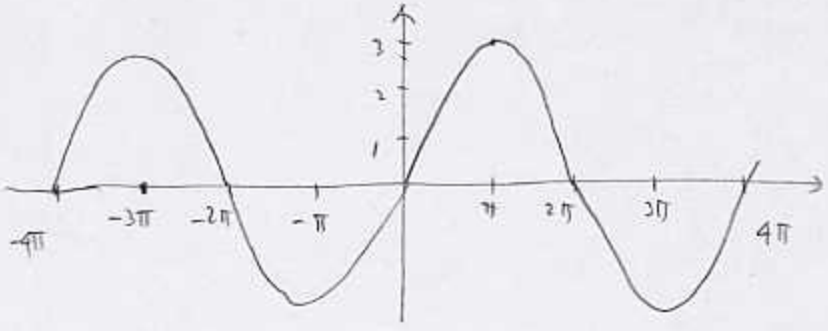
$\log_2(4x^2 - 16x + 16) - 2\log_2(x-2) = \log_2\left(4(x^2-4x+4)\right) - \log_2(x-2)^2$

$= \log_2 \frac{4(x^2-4x+4)}{(x-2)^2} = \log_2 \frac{4\cdot(x-2)^2}{(x-2)^2} = \log_2 4 = 2$

9. (4pts) Draw two periods of the graph of $y = 3\sin\left(\frac{1}{2}x\right)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).

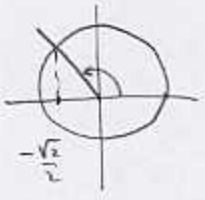
amplitude $= 3$

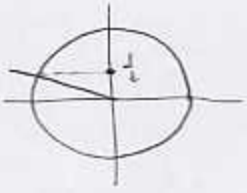
period $= 2\pi \cdot \frac{1}{\frac{1}{2}} = 4\pi$

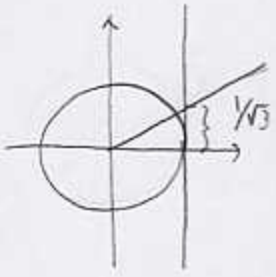


10. (6pts) Without using the calculator, find the exact values of the following expressions. Draw the unit circle and the appropriate angle under the expression.

$\cos 135° = -\frac{\sqrt{2}}{2}$ $\qquad \sin\frac{5\pi}{6} = \frac{1}{2}$ $\qquad \arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}$







11. (5pts) If $\sin\theta = -\frac{1}{4}$ and θ is in the fourth quadrant, find $\tan\theta$ and $\sin(2\theta)$.

$\sin\theta = -\frac{1}{4} = \frac{y}{r}$

$y = -1$

$r = 4$

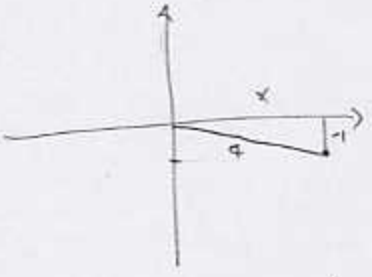
$x^2 + (-1)^2 = 4^2$

$x^2 = 16 - 1$

$x = \pm\sqrt{15}$

$x = \sqrt{15}$

(angle in quad. 4)



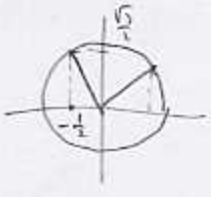
$\tan\theta = \frac{y}{x} = \frac{-1}{\sqrt{15}}$

$\sin(2\theta) = 2\sin\theta\cos\theta = 2\cdot\left(-\frac{1}{4}\right)\cdot\frac{\sqrt{15}}{4} = -\frac{\sqrt{15}}{8}$

12. (4pts) Use an addition formula to find the exact value of $\cos 165°$.

$\cos 165° = \cos(120° + 45°) = \cos 120°\cos 45° - \sin 120°\sin 45°$

$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{-\sqrt{2}-\sqrt{6}}{4} = -\frac{\sqrt{6}+\sqrt{2}}{4}$



13. (6pts) How many milliliters of a 15% solution of muriatic acid needs to be added to 100ml of a 30% solution in order to get a 20% solution? Don't forget to write down what your variable means.

x ml, 15%	+	100 ml, 30%	=	x+100 ml, 20%

pure acid: $0.15x + 0.3 \cdot 100 = 0.2(x+100)$

$0.15x + 30 = 0.2x + 20$

$10 = 0.05x$

$\frac{10}{0.05} = x$

$x = 200$ ml

14. (6pts) The population of the southwestern town Hukapsitti follows the exponential law. Five years ago, there were 35,000 inhabitants in the town. Currently, 79,000 people live there.

a) Write the function that describes the population of the town.

b) How many people will live in Hukapsitti in 3 years?

a) $P = P_0 e^{kt}$ (in thousands)

$P_0 = 35$

$79 = P(5) = 35e^{k \cdot 5}$

$79 = 35e^{5k}$

$\frac{79}{35} = e^{5k}$ / ln

$\ln \frac{79}{35} = 5k$

$k = \frac{1}{5} \ln \frac{79}{35} \approx 0.1628$

b) Need $P(8) = 35e^{k \cdot 8} = 128.75$

About 128,750 people

Bonus (7pts) Consider the rational function $f(x) = \dfrac{(x+2)(x-3)}{x^2-16}$.

a) Find the domain of f and the vertical asymptotes of the graph.

b) Find the x-intercepts of the graph and the y-intercept.

c) Find any horizontal asymptotes that the graph may have.

d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.

a) Domain: $x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$

D: all reals except ± 4

vertical asymp at ± 4

b) $(x+2)(x-3) = 0$ $\quad$ y-int $= \frac{-6}{-16} = \frac{3}{8}$

$x = -2, 3$

c) $\deg(x+2)(x-3) = 2$

$\deg(x^2-16) = 2$

so $y = \frac{1}{1}$ is a horizontal asymptote