Fall '05/MAT 150/Exam 4 **Name:** Solution *Show all your work.*

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \sin(2\theta) = 2\sin\theta\cos\theta \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \qquad \begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta} \qquad \tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

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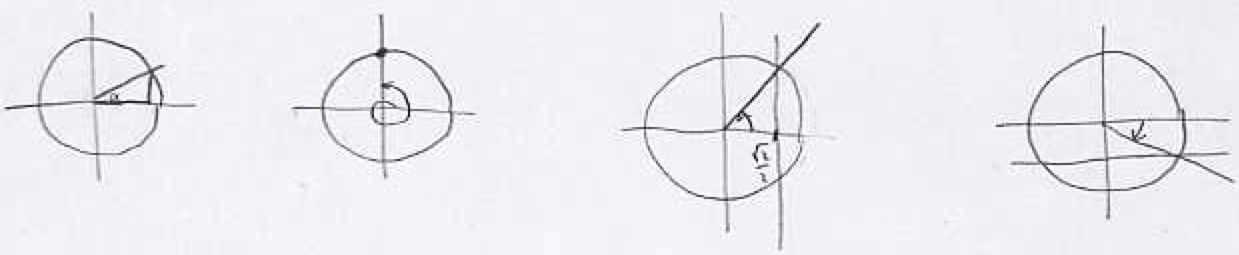
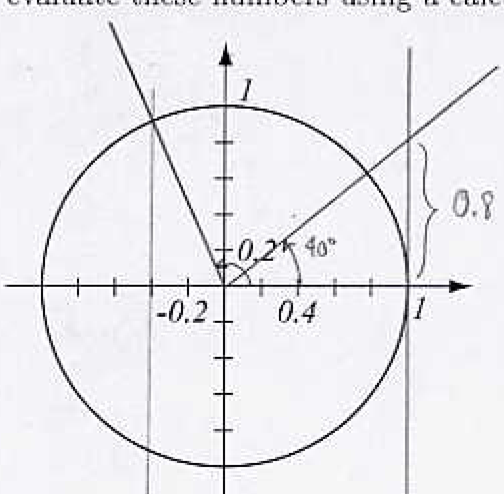
1. (8pts) Without using the calculator, find the exact values of the following expressions. Draw the unit circle and the appropriate angle under the expression.

$\cos 30° = \frac{\sqrt{3}}{2}$ $\qquad$ $\tan\frac{5\pi}{2} = \frac{1}{0}$ $\qquad$ $\arccos\frac{\sqrt{2}}{2} = \frac{\pi}{4}$ $\qquad$ $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

not defined



2. (4pts) Use the picture below to estimate $\tan 40°$ and $\arccos(-0.4)$ (in degrees). Then evaluate these numbers using a calculator and compare your answers.

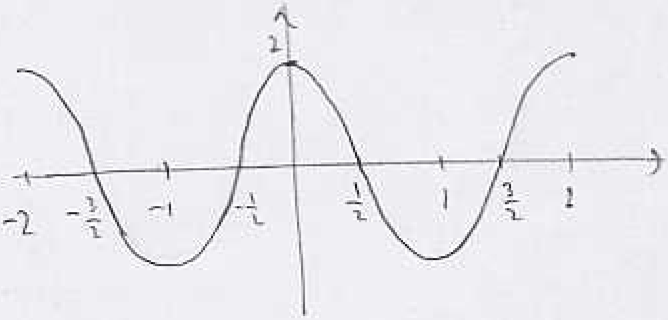


	calculator
$\tan 40° \approx 0.8$	0.84
$\arccos(-0.4) \approx 110°$	113.58°

3. (4pts) Draw two periods of the graph of $y = 2\cos(\pi x)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).

Amplitude = 2

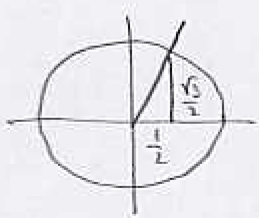
Period = $2\pi \cdot \frac{1}{\pi} = 2$



4. (4pts) Use an addition formula to find the exact value of $\cos 105°$.

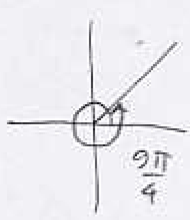
$$\cos 105° = \cos(60° + 45°) = \cos 60° \cos 45° - \sin 60° \sin 45°$$

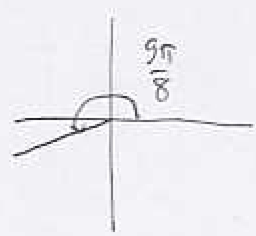
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$$



5. (4pts) Use a double-angle formula to find the exact value of $\sin \frac{9\pi}{8}$.

$$\sin^2 \frac{9\pi}{8} = \frac{1 - \cos\left(2 \cdot \frac{9\pi}{8}\right)}{2} = \frac{1 - \cos\left(\frac{9\pi}{4}\right)}{2} = \frac{1 - \frac{\sqrt{2}}{2}}{2} \cdot \frac{2}{2} = \frac{2 - \sqrt{2}}{4}$$

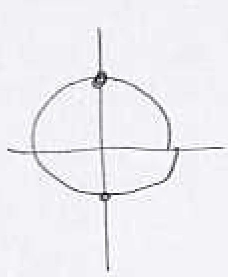




$$\sin \frac{9\pi}{8} = -\sqrt{\frac{2 - \sqrt{2}}{4}} = -\frac{\sqrt{2 - \sqrt{2}}}{2}$$

because it is in 3rd quadrant

6. (3pts) State the angles for which $\sec\theta$ is not defined. Explain. (Hint: looking at the unit circle and writing what $\sec\theta$ is in terms of x and y coordinates may help.)



$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{x}$$

undefined when $x = 0$

$$\theta = \frac{\pi}{2} + k \cdot 2\pi$$

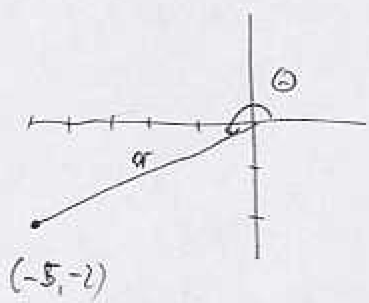
or $\theta = \frac{3\pi}{2} + k \cdot 2\pi$

7. (4pts) Show the identity: $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$.

$$\sec^4\theta - \sec^2\theta = \sec^2\theta(\sec^2\theta - 1) = \sec^2\theta \tan^2\theta =$$

$$= (\tan^2\theta + 1)\tan^2\theta = \tan^4\theta + \tan^2\theta$$

8. (6pts) If $\tan\theta = \frac{2}{5}$ and θ is in the third quadrant, find $\sin(2\theta)$ and $\cos\frac{\theta}{2}$.



$\tan\theta = \frac{y}{x} = \frac{2}{5}$

$y = -2$

$x = -5$

$r^2 = (-2)^2 + (-5)^2$

$= 4 + 25$

$r = \sqrt{29}$

$\sin\theta = -\frac{2}{\sqrt{29}}$

$\cos\theta = -\frac{5}{\sqrt{29}}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cdot\left(-\frac{2}{\sqrt{29}}\right)\left(-\frac{5}{\sqrt{29}}\right) = \frac{20}{29}$

$\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2} = \frac{1 - \frac{5}{\sqrt{29}}}{2}\cdot\frac{\sqrt{29}}{\sqrt{29}}$

$= \frac{\sqrt{29} - 5}{2\sqrt{29}}$

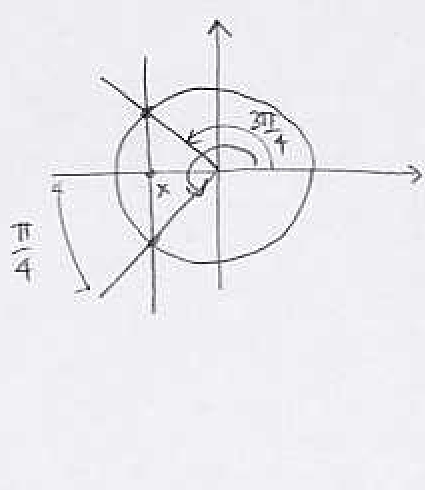
$\cos\frac{\theta}{2} = -\sqrt{\frac{\sqrt{29}-5}{2\sqrt{29}}}$ since $\frac{\theta}{2}$ is in quadrant 2

9. (4pts) Find the exact values of the expressions below. Draw a picture if helpful and do not use the calculator.

$\tan(\arctan 4.13) =$ 4.13

$\arccos\Big(\underbrace{\cos\left(\frac{5\pi}{4}\right)}_{x}\Big) = \arccos x$

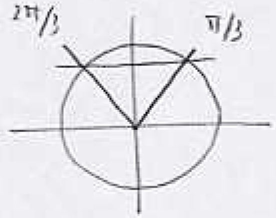
$= \frac{3\pi}{4}$



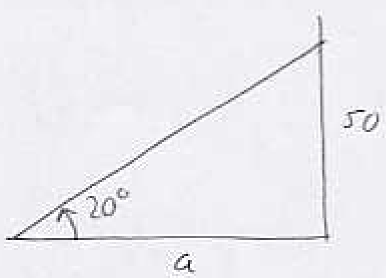
10. (4pts) Find all the solutions of the equation $2\sin\theta - \sqrt{3} = 0$.

$2\sin\theta = \sqrt{3}$ $\qquad \theta = \frac{\pi}{3} + k \cdot 2\pi$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\qquad \theta = \frac{2\pi}{3} + k \cdot 2\pi$



11. (5pts) Suppose you are headed toward a building 50 meters high. If the angle of elevation to the top of the building is 20°, how far away from the building are you?



$\frac{50}{a} = \tan 20°$

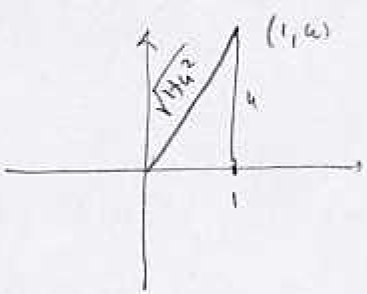
$\frac{50}{\tan 20°} = a$

$a = 137.38\,m$

Bonus (5pts) Show that $\sin(\arctan u) = \dfrac{u}{\sqrt{u^2+1}}$.

$\tan\theta = u = \frac{u}{1}$ $\qquad r^2 = 1^2 + u^2$

$r = \sqrt{1+u^2}$



$\sin\theta = \frac{y}{r} = \frac{u}{\sqrt{1+u^2}}$