Fall '05/MAT 150/Exam 3 **Name:** Solution *Show all your work.*

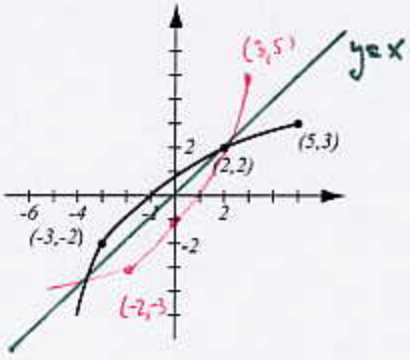
1. (4pts) Let $f(x) = x^2 + 3x - 1$ and $g(x) = x - 5$. Find $(f \circ g)(x)$ and simplify.

$(f \circ g)(x) = f(g(x)) = f(x-5) = (x-5)^2 + 3(x-5) - 1$

$= x^2 - 10x + 25 + 3x - 15 - 1$

$= x^2 - 7x + 9$

2. (4pts) The graph of f is given. Explain why f has an inverse and find the graph of its inverse function.



Function passes the horizontal line test, so it is one-to-one and hence has an inverse

3. (6pts) Solve the equations

$\log_x 8 = 2$

$x^2 = 8$

$x = \sqrt{8}$

$25^{x+2} = \left(\frac{1}{5}\right)^{3x-1}$

$(5^2)^{x+2} = (5^{-1})^{3x-1}$

$5^{2(x+2)} = 5^{-(3x-1)}$

$2x + 4 = -3x + 1$

$5x = -3$

$x = -\frac{3}{5}$

4. (4pts) Evaluate without using the calculator:

$\log_4 16 = 2$ $\log_2 \frac{1}{8} = -3$ $\ln \sqrt{e} = \frac{1}{2}$ $\log_5 \sqrt[3]{25} = \frac{2}{3}$

$4^2 = 16$ $2^{-3} = \frac{1}{8}$ $e^{\frac{1}{2}} = \sqrt{e}$ $5^{\frac{2}{3}} = \sqrt[3]{25} = \sqrt[3]{5^2} = 5^{\frac{2}{3}}$

5. (3pts) Write as a sum of logarithms. Express powers as factors. Simplify if possible.

$\log_2(2^x(x+1)^3) = \log_2 2^x + \log_2 (x+1)^3$

$= x + 3\log_2(x+1)$

6. (3pts) Write as a single logarithm. Simplify if possible.

$\ln(x^2+7x+12) - 3\ln(x+4) = \ln(x^2+7x+12) - \ln(x+4)^3$

$= \ln \frac{x^2+7x+12}{(x+4)^3} = \ln \frac{(x+3)(x+4)}{(x+4)^3}$

$= \ln \frac{x+3}{(x+4)^2}$

7. (5pts) Solve the equation.

$\log_2(x+1) + \log_2(x+3) = 3$

$\log_2 (x+1)(x+3) = 3$

$x^2+4x+3 = 2^3$

$x^2+4x-5 = 0$

$(x+5)(x-1) = 0$

$x = -5$ or $x = 1$

not a solution since $-5+1 < 0$ so $\log_2(-5+1)$ is not defined

8. (7pts) The amount of carbon 14 in a specimen is given by $A(t) = A_0 e^{kt}$, where A_0 is the original amount of carbon 14.
a) Given that the half-life of carbon 14 is 5600 years, find what k is.
b) A fossilized leaf contains 70% of its original amount of carbon 14. How old is the fossil?

a) $\frac{1}{2}A_0 = A_0 e^{k \cdot 5600}$

$\frac{1}{2} = e^{k 5600} \quad | \ln$

$\ln \frac{1}{2} = k \cdot 5600$

$k = \frac{\ln \frac{1}{2}}{5600}$

≈ -0.0001238

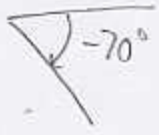
b) $0.7A_0 = A_0 e^{-0.0001238\, t}$

$0.7 = e^{-0.0001238 t} \quad | \ln$

$\ln 0.7 = -0.0001238 t$

$t = \frac{\ln 0.7}{-0.0001238} \approx 2881.61$ years

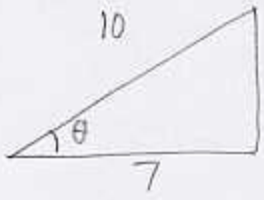
9. (2pts) Roughly sketch angles of measure $-70°$ and $\frac{3\pi}{5}$ radians.



10. (3pts) Mars makes one revolution in 1447 minutes. What is its angular speed in radians per second?

$\frac{1 \text{ rev}}{1447 \text{ min}} = \frac{2\pi \text{ rad}}{1447 \cdot 60 \text{ s}} = \frac{2\pi}{86820}$ rad/s $= 7.237 \times 10^{-5}$ rad/s

11. (5pts) In a right triangle, the leg adjacent to θ has length 7 and the hypothenuse has length 10. Find $\sin\theta$, $\cot\theta$ and $\sec\theta$.



$b = \sqrt{51}$

$7^2 + b^2 = 10^2$

$b^2 = 100 - 49 = 51$

$b = \sqrt{51}$

$\sin\theta = \frac{\sqrt{51}}{10}$

$\cot\theta = \frac{7}{\sqrt{51}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{7}{10}} = \frac{10}{7}$

12. (4pts) You are running on a circular path of radius 100m. If you have swept an angle of 105°, what distance have you run? (Hint: convert to radians.)

$105^\circ = 105 \cdot \frac{\pi}{180}$ radians $= \frac{7}{12}\pi$ radians

$s = r\theta = 100 \cdot \frac{7}{12}\pi = \frac{175}{3}\pi$ m

$= 183.26$ m

Bonus (5pts) Let $f(x) = 17 + 4e^{x-3}$. Find the formula for the inverse of this function.

$y = 17 + 4e^{x-3}$

$y - 17 = 4e^{x-3}$

$\frac{y-17}{4} = e^{x-3} \quad | \ln$

$\ln\frac{y-17}{4} = x - 3$

$x = 3 + \ln\frac{y-17}{4}$