Fall '05/MAT 150/Exam 2 **Name:** Solution *Show all your work.*

1. (5pts) Let $f(x) = x^2 + 3$ and $g(x) = x - 1$. Find the following:

$f(2) =$ $2^2+3 = 7$

$g(3t+4) =$ $3t+4-1 = 3t+3$

$(f \cdot g)(x) =$ $(x^2+3)(x-1) = x^3 - x^2 + 3x - 3$

2. (10pts) Use the graph of the function f, below, to answer the following questions.

a) What is $f(3)$? $f(3) = -3$

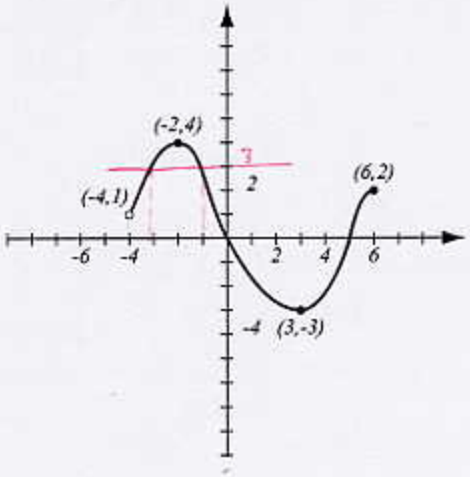
b) What are the x-intercepts?

c) Where is the function increasing?

d) Where does f have a local maximum? What is its value?

e) What are the solutions of the equation $f(x) = 3$?

f) What is the domain of the function?



a) $f(3) = -3$

b) $x = 0$, $x = 5$

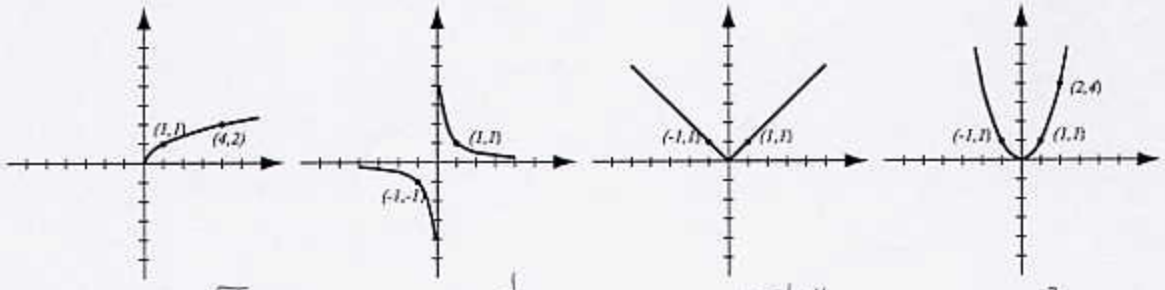
c) on $(-4, -2)$ and $(3, 6)$

d) at $x = -2$ with value $y = 4$

e) Approx $x = -3.2$ and $x = -1$

f) Domain: $(-4, 6]$

3. (4pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = \sqrt{x}$ $y = \frac{1}{x}$ $y = |x|$ $y = x^2$

4. (7pts) The quadratic function $f(x) = x^2 + 2x + 5$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

d) What is the range of the function?

a) $x^2+2x+5=0$

$x = \frac{-2 \pm \sqrt{4-4 \cdot 5}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$

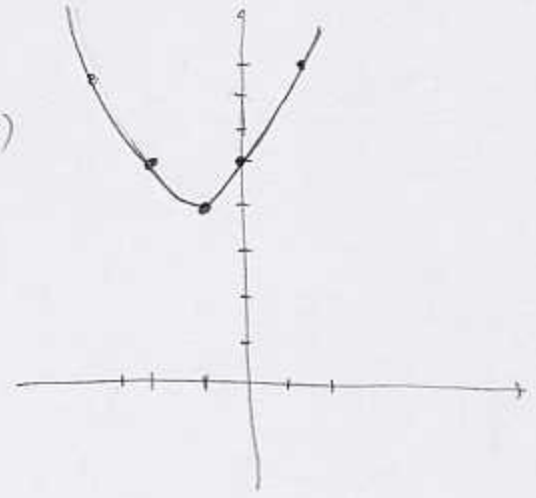
no real solutions

so no x-int.

b) $x = -\frac{2}{2 \cdot 1} = -1$

$y = (-1)^2 + 2(-1) + 5 = 4$

c)



x	y
1	8
-3	8

d) Range $= [4, \infty)$

5. (4pts) Find the domain of the function $f(x) = \frac{3}{\sqrt{5-2x}}$

Must have

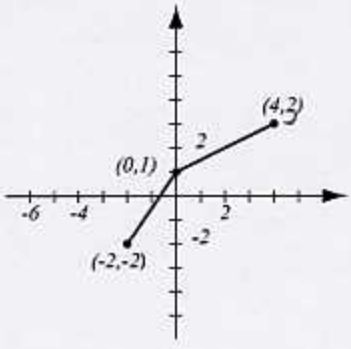
$5 - 2x > 0$ (can't be zero because it is in denominator)

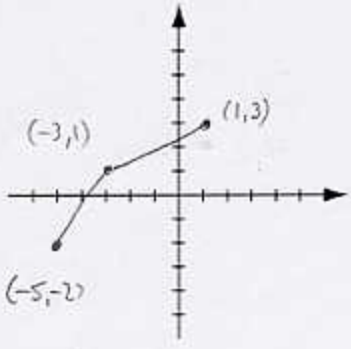
$5 > 2x$

$\frac{5}{2} > x$

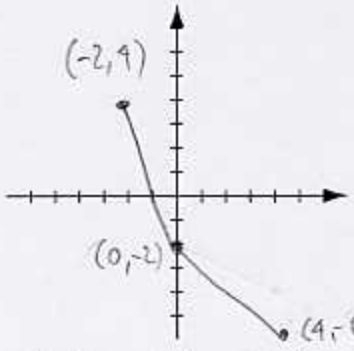
$D = (-\infty, \frac{5}{2})$

6. (5pts) The graph of $f(x)$ is drawn below. Find the graphs $f(x+3)$ and $-2f(x)$ and label all the relevant points.





shift left 3



stretch vertically by factor 2 then reflect in x-axis

7. (8pts) Consider the polynomial $P(x) = x^4 - 3x^3 + x + 2$. Answer the following (decimal answers should have accuracy to two decimal places).

a) Find the x-intercepts of the graph and the y-intercept.

b) P behaves like what function for large $|x|$?

c) Find the smallest turning point of P.

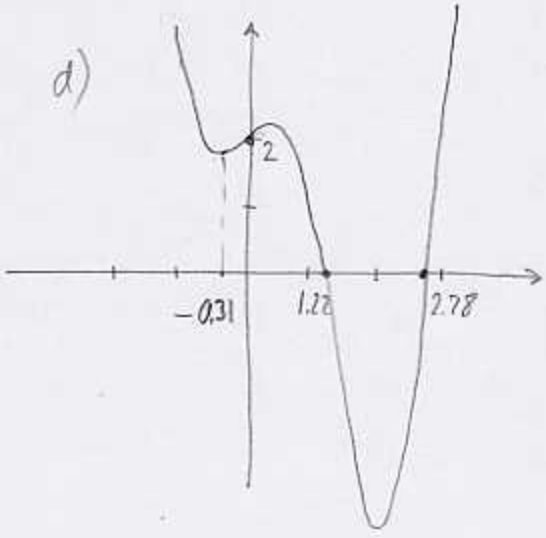
d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.

a) $x = 1.22$ $y\text{-int} = 2$
or $x = 2.78$

b) P behaves like x^4

c) Local min at -0.31 with value 1.79

d)



8. (7pts) The price p and the quantity x sold of a certain product obey the demand equation $p = -\frac{1}{3}x + 100$, $0 \leq x \leq 300$.
a) Express the revenue R as a function of x.
b) What quantity maximizes revenue? What is the maximal revenue?
c) What price should the company charge to maximize revenue?

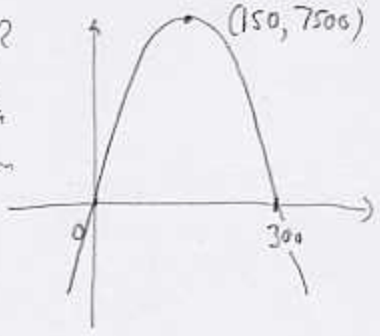
a) $R = x \cdot p = x\left(-\frac{1}{3}x + 100\right) = -\frac{1}{3}x^2 + 100$

c) $p = -\frac{1}{3} \cdot 150 + 100$

$= \$50$

b) Graph of R is a parabola opening down

x-int. are $x = 0$ and $x = 300$



Maximal revenue is 7500.

Bonus (5pts) The Crooncard company makes talking greeting cards. To wholesalers they charge \$1.25 per card for any number of cards up to 200. An order for more than 200 cards is priced as \$250 plus \$1.10 for every card in excess of 200.
a) Write the piecewise-defined function that describes the price P as a function of the number of cards x bought.
b) Sketch the graph of the function.

a) $$P(x) = \begin{cases} 1.25x & \text{if } 0 \leq x \leq 200 \\ 250 + 1.10(x - 200) & \text{if } 200 < x \end{cases}$$

x	$P(x)$
0	0
200	250

x	$P(x)$
200	250
400	470