Fall '04/MAT 250/Worksheet 3 **Name:** Solution *Show all your work.*

1. (7pts) Use L'Hospital's rule to find the limits:

$$\lim_{x\to 0} \frac{\sin x - x}{x^3} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x - 1}{3x^2} \overset{L'H}{=} \lim_{x\to 0} \frac{-\sin x}{6x} \overset{L'H}{=} \lim_{x\to 0} \frac{-\cos x}{6} = -\frac{1}{6}$$

$\frac{0}{0}$ $\quad \frac{1-1}{0} = \frac{0}{0}$ $\quad \frac{0}{0}$

$$\lim_{x\to\infty} (x^2+x+1)^{\frac{1}{x}} = e^0 = 1$$

$y = (x^2+x+1)^{\frac{1}{x}}$

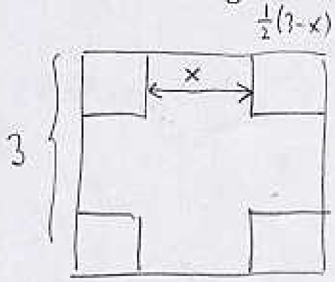
$\ln y = \frac{1}{x} \ln(x^2+x+1)$

$$\lim_{x\to\infty} \frac{\ln(x^2+x+1)}{x} = \lim_{x\to\infty} \frac{\frac{2x+1}{x^2+x+1}}{1} = \lim_{x\to\infty} \frac{2x+1}{x^2+x+1} \overset{L'H}{=}$$

$\frac{\ln(\infty+\infty+1)}{\infty} = \frac{\infty}{\infty}$ $\quad \frac{\infty}{\infty}$

$$= \lim_{x\to\infty} \frac{2}{2x+1} = \frac{2}{\infty} = 0$$

2. (8pts) A box with an open top is to be constructed from a square piece of cardboard, 3ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.



$V = x^2 \cdot \frac{1}{2}(3-x) = \frac{1}{2}(3x^2 - x^3)$

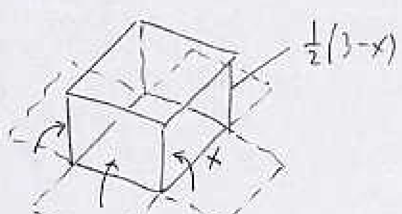
Job: maximize V for $0 \le x \le 3$

$V'(x) = \frac{1}{2}(6x - 3x^2)$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

$x = 0$ or $x = 2$



x	$V(x)$	
2	$\frac{1}{2} \cdot 4 \cdot 1 = 2$	← max volume
0	0	
3	0	