Fall '04/MAT 250/Worksheet 2 **Name:** Solution *Show all your work.*

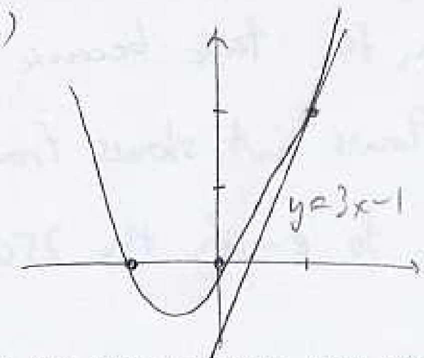
1. (6pts) The curve $y = x^2 + x$ is given.
a) Find the equation of the tangent line to the curve at point $(1, 2)$. (Use a limit to find its slope.)
b) Sketch the curve and the line you find in a) and verify that the line is indeed tangent to the curve.

a) $a = 1$, $f(1) = 2$
$f(x) = x^2 + x$

$$m = \lim_{x\to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{x^2 + x - 2}{x - 1} =$$
$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{x-1}} = 1 + 2 = 3$$

$y - 2 = 3(x - 1)$

$y = 3x - 1$



2. (9pts) It takes one hour to drain, from the bottom, a cylindrical tank holding 1000 liters of water. According to Torricelli's law, the volume of water remaining in the tank is $V(t) = 1000(1 - t)^2$, where V is in liters and t in hours, $0 \le t \le 1$.
a) Find the instantaneous rate at which the water is draining at $t = \frac{1}{2}$. (Use an appropriate limit). Why is the rate negative?
b) What is $V(\frac{1}{2})$? If water were to continue draining at the same rate as you found in a), at what time t would the tank be empty?
c) Explain why it will take longer than you found in b) to actually drain the tank (an hour). A graph of V might help.

a) Inst. rate at $t = \frac{1}{2}$ $= \lim_{t \to \frac{1}{2}} \frac{V(t) - V(\frac{1}{2})}{t - \frac{1}{2}} = \lim_{t \to \frac{1}{2}} \frac{1000(1-t)^2 - 1000(\frac{1}{2})^2}{t - \frac{1}{2}}$

$$= \lim_{t \to \frac{1}{2}} \frac{1000\left((1-t)^2 - (\frac{1}{2})^2\right)}{t - \frac{1}{2}} = 1000 \lim_{t \to \frac{1}{2}} \frac{(1 - t - \frac{1}{2})(1 - t + \frac{1}{2})}{t - \frac{1}{2}} = 1000 \lim_{t \to \frac{1}{2}} \frac{\overbrace{(\tfrac{1}{2} - t)}^{= -(t - \frac{1}{2})}(\frac{3}{2} - t)}{(t - \frac{1}{2})}$$

$$= 1000 \lim_{t \to \frac{1}{2}} -(\tfrac{3}{2} - t) = -1000(\tfrac{3}{2} - \tfrac{1}{2}) = -1000 \ \ell/\text{hr}$$

Rate is negative because volume of water is decreasing (change is neg. and so is rate of change)

b) $V(\frac{1}{2}) = 1000\left(1-\frac{1}{2}\right)^2 = 1000 \cdot \frac{1}{4} = 250\,\ell$

If water were to continue draining at rate $-1000\,\ell/hr$

$250\,\ell$ would empty in $\frac{250}{1000} = \frac{1}{4}$ hr

Hence, tank would be empty at $t = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ hr.

c) It takes an hour to drain the tank because the rate at which water flows out slows from $-1000\,\ell/min$, so it takes more time to empty the $250\,\ell$.