Fall '04/MAT 250/Exam 4 Name: Solution *Show all your work.*

1. (5pts) Find f if $f'(x) = \frac{1}{x} + 5x$ and $f(2) = 4$.

$f'(x) = \frac{1}{x} + 5x$

$f(x) = \ln|x| + 5\frac{x^2}{2} + C$

$4 = f(2) = \ln 2 + 5 \cdot 2 + C$

$-6 - \ln 2 = C$

$f(x) = \ln|x| + \frac{5}{2}x^2 - 6 - \ln 2$

2. (10pts) Evaluate using the Fundamental Theorem of Calculus, part 2:

a) $\int_1^4 \frac{1}{\sqrt{x}}\,dx = \int_1^4 x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\Big|_1^4 = 2\sqrt{x}\Big|_1^4 = 2(2-1) = 2$

b) $\int_0^{\ln 5} e^{3x}\,dx = \frac{e^{3x}}{3}\Big|_0^{\ln 5} = \frac{1}{3}\left(\underbrace{e^{3\cdot\ln 5}}_{(e^{\ln 5})^3} - e^0\right) = \frac{1}{3}(125-1) = \frac{124}{3}$

3. (3pts) Write as a single integral:

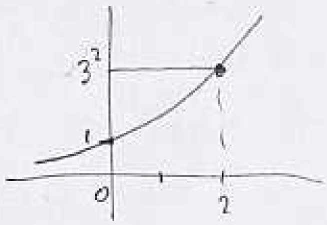
$\int_1^5 f(x)\,dx - \int_1^3 f(x)\,dx + \int_5^7 f(x)\,dx = \int_3^5 f(x)\,dx + \int_5^7 f(x)\,dx = \int_3^7 f(x)\,dx$

4. (2pts) Simplify using part 1 of the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\int_0^x \tan t\,dt = \begin{bmatrix} \text{FTC} \\ \text{part 1} \end{bmatrix} = \tan x$$

5. (4pts) Use properties of integrals to show that $2 \le \int_0^2 3^x\,dx \le 18$.

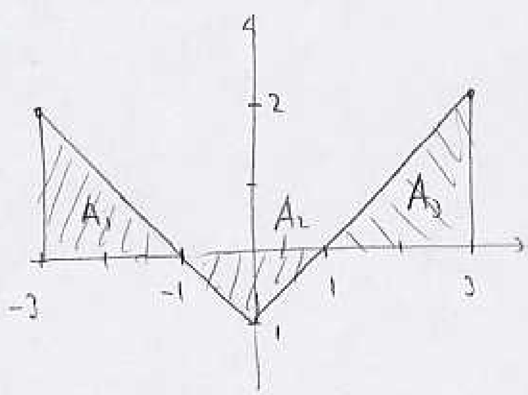
On $[0,2]$ $\quad 3^0 \le 3^x \le 3^2$



so $\quad 1\cdot(2-0) \le \int_0^2 3^x \le 9\cdot(2-0)$

$$2 \le \int_0^2 3^x\,dx \le 18$$

6. (5pts) Use the "area" interpretation of the integral to find $\int_{-3}^{3}(|x|-1)\,dx$. Draw a picture.



$$\int_{-3}^{3}(|x|-1)\,dx = A_1 - A_2 + A_3$$

$$= \frac{1}{2}2\cdot 2 - \frac{1}{2}2\cdot 1 + \frac{1}{2}2\cdot 2$$

$$= 3$$

Fall '04/MAT 250/Exam 4, take-home portion Name: *Show all your work.*

The rules: you may use your book and notes on this take-home quiz. Your work is to be entirely your own: you may not talk to anybody else about the quiz problems. Turn the exam in on Thursday, Dec. 9th.

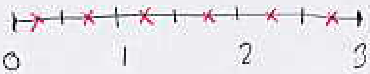
7. (9pts) Let $f(x) = x^2 - x$.

a) Find the Riemann sum for f on $[0, 3]$ using 6 subintervals and midpoints as sample points.

b) Draw the graph of the function with the appropriate rectangles (big and beautiful, okay?) and state what the Riemann sum in a) represents.

c) Compute $\int_0^3 (x^2 - x)\,dx$ using the Fundamental Theorem of Calculus. Is the number in a) close to the actual integral?

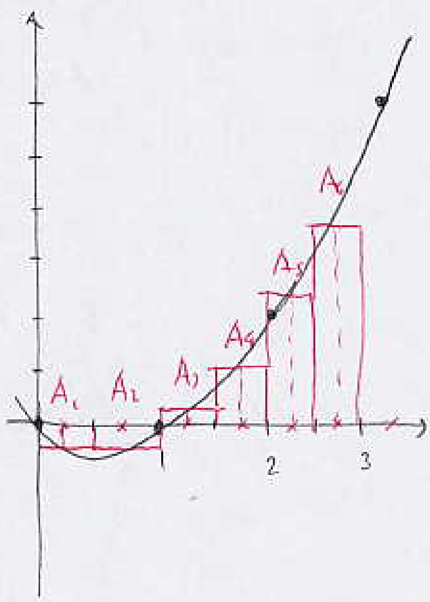
a) $\Delta x = \frac{1}{2}$



$$\sum_{i=1}^{6} f(x_i^*)\,\Delta x = \left(f(\tfrac{1}{4}) + f(\tfrac{3}{4}) + f(\tfrac{5}{4}) + f(\tfrac{7}{4}) + f(\tfrac{9}{4}) + f(\tfrac{11}{4})\right)\tfrac{1}{2}$$

$$= (-0.1875 + (-0.1875) + 0.3125 + 1.3125 + 2.8125 + 4.8125)\cdot\tfrac{1}{2}$$

$$=$$

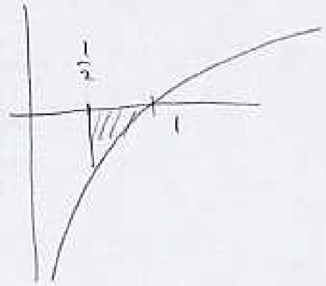


b) Riemann sum is $-A_1 - A_2 + A_3 + A_4 + A_5 + A_6$

c) $$\int_0^3 (x^2 - x)\,dx = \frac{x^3}{3} - \frac{x^2}{2}\Big|_0^3 = \frac{1}{3}(27-0) - \frac{1}{2}(9-0)$$

$$= 9 - \frac{9}{2} = \frac{9}{2} = 4.5$$

8. (3pts) Explain how you know without evaluating anything that $\int_{1/2}^{1} \ln x\,dx \le 0$.



on $[\frac{1}{2}, 1]$

$\ln x \le 0$

so $\int_{1/2}^{1} \ln x\,dx \le 0$.

9. (4pts) Write in sigma notation.

$$\frac{1}{3}-\frac{2}{4}+\frac{3}{5}-\frac{4}{6}+\cdots+\frac{21}{23}-\frac{22}{24}= \quad \sum_{i=1}^{22}(-1)^{i}\frac{i}{i+2}$$

10. (5pts) This problem is about the integral $\int_{-1}^{2}\frac{1}{x^2}\,dx$.
a) Explain why you cannot use the Fundamental Theorem of Calculus to compute it.
b) Ignore a) for a moment and compute the integral using the FTC anyway. You should get a negative number. Looking at the function, how do you know your answer cannot be correct (hence, something is wrong with applying the FTC)?

a) $\frac{1}{x^2}$ is not continuous on $[-1,2]$

b) $$\int_{-1}^{2} x^{-2}\,dx = \left.\frac{x^{-1}}{-1}\right|_{-1}^{2} = \left.-\frac{1}{x}\right|_{-1}^{2} = -\left(\frac{1}{2}-\frac{1}{-1}\right) = -\frac{3}{2}$$

$\frac{1}{x^2}\geq 0$ so we should have gotten $\int_{-1}^{2}\frac{1}{x^2}\,dx \geq 0$

Bonus. (5pts) A broken bone a day keeps the doctor in pay! At time $t = 2$ we find Wile E. Coyote at height 40m above ground and ascending vertically at velocity 15m/s. Use antiderivatives to find his position function. What is the highest altitude he reaches? Assume the acceleration of gravity is a constant 10m/s^2.

$a(t) = -10$

$v(t) = -10t + C$

$15 = v(2) = -20 + C$

$35 = C$

$v(t) = -10t + 35$

$s(t) = -10\frac{t^2}{2} + 35t + D$

$40 = s(2) = -10\cdot 2 + 35\cdot 2 + D$

$-10 = D$

$s(t) = -5t^2 + 35t - 10$

Highest altitude reached when $v(t) = 0$

$-10t + 35 = 0$

$t = 3.5$

$s(3.5) = 51.25$ m