

The rules: you may use your book and notes on this take-home quiz. Your work is to be entirely your own: you may not talk to anybody else about the quiz problems. Turn the quiz in on Monday, Oct 25th.

Differentiate and simplify where appropriate:

$$1. (4pts) \frac{d}{dx} \sqrt{xe^x} = \frac{1}{2\sqrt{xe^x}} (1 \cdot e^x + xe^x) = \frac{e^x(1+x)}{2\sqrt{xe^x}}$$

$$2. (4pts) \frac{d}{dx} \frac{\sin^2 x}{\cos x} = \frac{2\sin x \cos x \cos x - \sin^2 x (-\sin x)}{\cos^2 x} = \frac{2\sin x \cos^2 x + \sin^3 x}{\cos^2 x}$$

$$= \frac{\sin x (2\cos^2 x + \sin^2 x)}{\cos^2 x} = \sin x (2 + \tan^2 x)$$

$$3. (4pts) \frac{d}{dx} \sin \frac{1}{x^3 - 4x^2 + 1} = \cos \frac{1}{x^3 - 4x^2 + 1} \cdot (-1)(x^3 - 4x^2 + 1)^{-2} \cdot (3x^2 - 8x)$$

$$= - \frac{3x^2 - 8x}{(x^3 - 4x^2 + 1)^2} \cdot \cos \frac{1}{x^3 - 4x^2 + 1}$$

4. (4pts) The curve $x^2 + 2xy - y^2 + x = 2$ is given.

a) Show that the point (1, 2) is on the curve.

b) Find the equation of the tangent line to the curve at point (1, 2).

$$a) 1^2 + 2 \cdot 1 \cdot 2 - 2^2 + 1 = 1 + 1 = 2 \quad \text{ok}$$

$$b) x^2 + 2xy - y^2 + x = 2 \quad \frac{d}{dx}$$

$$2x + 2(1 \cdot y + x \cdot y') - 2yy' + 1 = 0$$

$$y'(2x - 2y) = -2x - 1 - 2y$$

$$y' = - \frac{2x + 1 + 2y}{2x - 2y}$$

$$x=1 \quad y=2 \quad y' = - \frac{7}{-2} = \frac{7}{2}$$

$$y - 2 = \frac{7}{2}(x - 1)$$

$$y = \frac{7}{2}x - \frac{3}{2} \quad \text{eq. of tangent line}$$

5. (4pts) We never actually proved that the formula for differentiating $\sqrt[3]{x}$ holds. Do this now in the same way we found the derivative of $\arcsin x$: First, let $y = \sqrt[3]{x}$, then $y^3 = x$. Differentiate this equation implicitly and find what y' is, keeping in mind that your answer needs to be in terms of x . Find the derivative of $\sqrt[3]{x}$ in the usual way and compare your answers — they should agree.

$$y = \sqrt[3]{x}$$

$$y^3 = x \quad \left| \frac{d}{dx} \right.$$

$$3y^2 y' = 1$$

$$y' = \frac{1}{3y^2} = \frac{1}{3(\sqrt[3]{x})^2} = \frac{1}{3x^{2/3}}$$

$$y = x^{1/3}$$

$$y' = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}}$$

← Same →

Bonus. (2pt) Use the grade computer on the website to answer the following. Take into account only grades we had before this quiz and assume you do not have attendance bonus points.

- What is your current average in the course?
- What average would you like to have after the second exam? What is the lowest score you need on the second exam to have this average?
- What is the lowest score you need on exam 3 in order to have the desired average from b), assuming on exam 2 you got the score from the second part of b)?